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Calculations for beams of infinite length with elastic supports,

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1. Introduction.

The attention of research workers has often been drawn to the problems relating to static calculations for a beam supported on elastic supports (1). KOITER (2) has given the most efficient method of calculating the stresses for a beam of *finite* length under gradually applied loads.

Beams of this sort are encountered in railway practice in the shape of running rails supported on cross sleepers. It will be agreed that such a beam may be considered as being of finite length and that the distance between the two elastic supports is constant. The sleepers will therefore be considered as springs whose stiffness is proportional to the vertical forces which are applied to them. The problem is thus that of a beam of infinite length supported on equally spaced springs.

The calculations are very considerably simplified if it is assumed that the beam is of infinite length. In this case it is only necessary to solve the problem for a beam which is acted upon by a force concentrated at a fixed point between any two supports. From the results thus obtained it is possible to deduce the deflection and the bending moments for a beam loaded in a specified manner. The first problem (which will be treated in Chapters 3 and 4) is relatively simple (3).

A simple calculation (4) for the limiting value can be obtained if the intermittent support in the form of a spring is considered as being replaced by a con-

(1) C. B. BIEZENO : Zeichnerische Ermittlung der elastischen Linie eines federnd gestützten, statische unbestimmten Balken. *Zeitschrift für angewandte Mathematik und Mechanik*, No. 4 (1924), p. 93.

J. J. KOCH : Enige toepassingen van de leer der eigenfuncties op vraagstukken uit de toegepaste mechanica, *Dissertatie Delft* (Delft, 1929), p. 57.

(2) W. T. KOITER : Berekening van veerend gesteunde balken. *De Ingenieur,* No. 55 (1940), page W. 57.

(3) On completion of this article it was noticed that the problems dealt with had already been treated previously in this bulletin. (See M. R. DESPRETS : Etude sur le calcul des rails, *Bulletin of the International Railway Congress Association, No. 5* [1921], p. 507.) However it is hoped that this additional contribution to the problem will assist in making the necessary calculations.

(4) H. ZIMMERMANN : Die Berechnung des Eisenbahn-Oberbaues (Berlin, 1888).

tinuous elastic bed. The method of this calculation is shown in Chapter 5 and in the following chapter the divergencies between deflections and bending moments obtained when treating the beam as being supported on equally spaced supports and those obtained when treating the beam as resting on a continuous elastic bed.

Chapter 7 shows the calculations necessary to calculate the deformations produced in a mercury contact threadle. Finally a concise method for making the calculations in the case of a beam of finite length will be given (Chapter 8).

These calculations are of equal utility in investigations concerning the theory of resistance of rails. Mr. DRIESSEN in the important set of investigations conducted by him, gives a practical method of calculation taking into account the result of numerous experiments (5).

2. The auxiliary problem.

Before discussing the main problem, it is proposed to investigate the beam shown in Fig. 1, where one of the ends is a vertical force D and a moment M, and where the other end is extended to infinity.

If the deflections of the beam at the elastic supports are shown as y_1, y_2 ... (see Fig. 1) and the elasticity of the support as c, then the resistant forces at these supports will be equal to cy_1, cy_2, etc.

Applying MAXWELL's equations, the deflection of free extremity y_1 and the angle of tangency φ_1 by $y_1 = \alpha D_1 + \beta M_1$, $\varphi_1 = \beta D_1 + \gamma M_1$. To simplify the calculations the equations can be written in the form

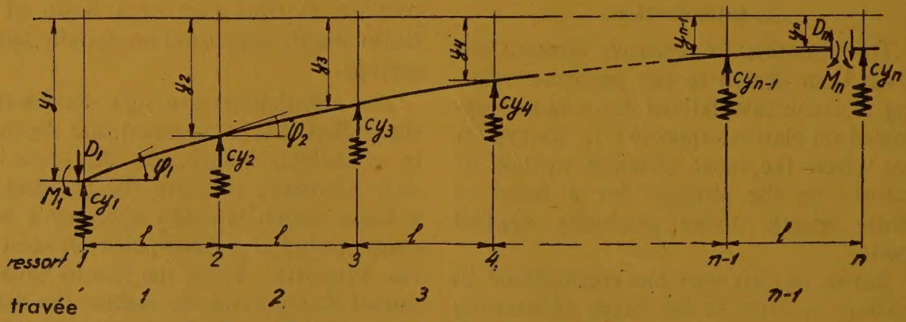

Fig. 1. — Beam referred to in the auxiliary problem.

Ressort = spring. — Travée = section.

$$\left.\begin{aligned} y_1 &= p\nu D_1 + q\frac{\nu}{l}M_1, \\ l\varphi_1 &= q\nu D_1 + r\frac{\nu}{l}M_1 \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (1)$$

in which

$$\nu = \frac{l^3}{6EI}; \quad . \quad . \quad . \quad . \quad (2)$$

(5) Ch. H. J. DRIESSEN : Nieuwe inzichten bij de berekening van de bovenbouw der spoorwegen, *De Ingenieur*, No. 51 (1936), page V. 75.

See also : M. P. KANDAOUROFF : Le calcul des éléments de la superstructure recommandé par l'Union des Administrations de chemins de fer de l'Europe centrale, *Bulletin of the International Railway Congress Association*, Volume 20 (1938), p, 961.

where E represents the modulus of elasticity of the material composing the beam, I the moment of inertia of the section with reference to an axis passing

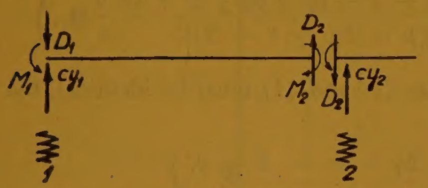

Fig. 2. — First section of the beam.

through the centre of gravity, l the distance between two adjacent supports. The object of this auxiliary calculation is to calculate the magnitudes p, q and r (which for ease in calculation are unreal numbers).

It is now necessary to consider the first element of the beam by supposing the beam to be cut immediately to the left of support No. 1. The shearing force and bending moment of this section are shown by D_2 and M_2. These can be easily ascertained by means of the formulae :

$$D_2 = D_1 - cy_1,\ M_2 = (D_1 - cy_1)l + M_1, \quad \ldots\ldots \quad (3)$$

and by the deformation equations, the relations

$$y_1 = y_2 + 2\nu(D_1 - cy_1) + 3\frac{\nu}{l}M_1,\ l\varphi_1 = l\varphi_2 + 3\nu(D_1 - cy_1) + 6\frac{\nu}{l}M_1. \quad (4)$$

Therefore, by analogy with (1), the beam being of infinite length

$$y_2 = p\nu D_2 + q\frac{\nu}{l}M_2,\ l\varphi_2 = q\nu D_2 + r\frac{\nu}{l}M_2. \quad \ldots\ldots \quad (5)$$

In the 6 equations (3), (4) and (5), y_2, φ_2, D_2 and M_2 are eliminated by substituting D_2 and M_2 of (3) in (5) and by replacing y_2 and φ_2 in (4) by the expressions thus found. If, for simplicity,

$$c\nu = \frac{cl^3}{6EI} = C \quad \ldots\ldots \quad (6)$$

[C being an unreal number (6)], the two equations are obtained

$$\{1 + (p + 2q + r + 2)C\}y_1 = (p + 2q + r + 2)\nu D_1 + (q + r + 3)\frac{\nu}{l}M_1,$$

$$(q + r + 3)Cy_1 + l\varphi_1 = (q + r + 3)\nu D_1 + (r + 6)\frac{\nu}{l}M_1,$$

(6) In order to give some indication of the size of C as far as it is applicable to railway sleepers, it may be stated that two equal loads of 7.5 metric tons, i.e. a total load of 15 metric tons have been placed on a sleeper at the point where the rail is carried. Under these conditions deflections were in the neighbourhood of 5 mm., so that for this sleeper $C = 3.10^4$ kgr./cm^2. When the sleepers are equally spaced at 60 cm. and the moment of inertia equals 3.360 cm^4 (which is the case for two rails weighing about 45 kgr./m.) the following values have been obtained : $E = 2{,}1.10^6$ kgr./cm^2, $\nu = 6.10^{-6}$ cm./kgr. and $C = 0.18$. This test took place on a yielding table. A deflection of about 0.5 mm. has also been measured on an extremely rigid table, so that if the sleeper spacing and the factor of rigidity of the rails do not vary, C has a value of 1.8.

from which the following are obtained :

$$\left.\begin{aligned} y_1 &= \frac{p + 2q + r + 2}{1 + (p + 2q + r + 2)C} \nu D_1 + \frac{q + r + 3}{1 + (p + 2q + r + 2)C} \frac{\nu}{l} M_1, \\ l\varphi_1 &= \frac{q + r + 3}{1 + (p + 2q + r + 2)C} \nu D_1 + \frac{r + 6 + (pr - q^2 + 6p + 6q + 2r + 3)C}{1 + (p + 2q + r + 2)C} \frac{\nu}{l} M_1. \end{aligned}\right\} \quad (7)$$

Being given the condition that the formulae (7) and (1) must be identical, the following equations are obtained :

$$\left.\begin{aligned} & p(p + 2q + r + 2)C - 2q - r - 2 = 0, \\ & q(p + 2q + r + 2)C - r - 3 = 0, \\ & \{(q + r)^2 - 6(p + q) - 3\}C - 6 = 0, \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (8)$$

in which p, q and r can be determined. The first two equations lead to

$$p(r + 3) = q(2q + r + 2) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

wherein C is eliminated. This formula will be used in subsequent calculations.

Equations (8) are easily solved by introduction of the auxiliary magnitudes

$$x = q + r, \; y = p + r, \; z = p + q \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

If (8) is substituted in the 1st. and 3rd. equations

$$p = \frac{y + z - x}{2}, \; q = \frac{z + x - y}{2}, \; r = \frac{x + y - z}{2}, \; . \; . \; . \quad (11)$$

it is found that

$$z(x + z + 2)C - 2x - 5 = 0, \qquad (x^2 - 6z - 3)C - 6 = 0; \; . \; . \; . \; . \quad (12)$$

these equations only having x and z in them. A relation which includes y can be obtained by substituting (11) in the second equation (8); however it is less complicated to effect this substitution in the formula

$$(p - q)(p + 2q + r + 2)C - 2q + 1 = 0,$$

which is obtained by subtracting the second equation (8) from the first equation (8) and thus is found

$$\{(x + z + 2)C + 1\}y = x(x + z + 2)C + x + z - 1 \; . \; . \; . \quad (13)$$

It will now be ascertained how x and z can be determined by means of the relation (12). If

$$z = \frac{1}{6}\left(x^2 - 3 - \frac{6}{C}\right) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

is introduced of the second equation (12) into the first equation (12) it becomes :

$$x^4 + 6x^3 + 6\left(1 - \frac{2}{C}\right)x^2 - 18\left(1 + \frac{6}{C}\right)x - 27 - \frac{216}{C} + \frac{36}{C^2} = 0 \quad . \quad (15)$$

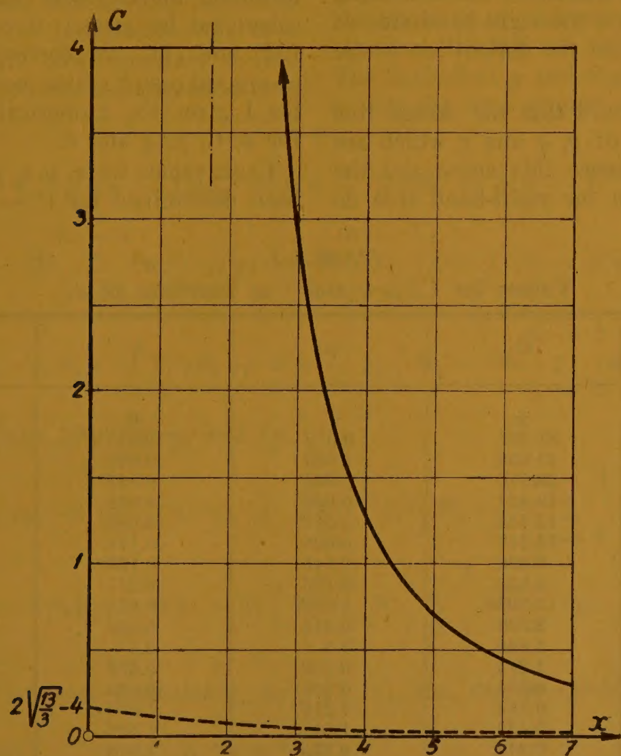

Fig. 3. — Graphical representation of the relationship between C and x.

In solving equation (15) limits due to physical considerations can be placed on the positive values of C and x. It will be found that if $x > 0$ the values of the terms on the left-hand side of equation (15) present a variation for $2\sqrt{\frac{13}{3}} - 4 < C < \infty$ and two variations for $0 < C < 2\sqrt{\frac{13}{3}} - 4$. Thus the equation (15) in the first case always has a positive root for x and in the second case 2 or no positive roots for x. Consequently, each value of C corresponding to a pair of values p, q and r, it is necessary to exclude the possibility of any positive root in the last case.

The value of C can be found directly from equation (15)

$$C = 6\,\frac{x + 6 \pm \sqrt{3(4x + 13)}}{(x^2 - 3)(x + 3)} \qquad (16)$$

as a function of x. The relation of x with reference to C can easily be shown graphically (Fig. 3). In the diagram the full line shows the curve corresponding to the $+$ sign on the right-hand side of equation (16) and the dotted line to the negative sign.

It will be found that the dotted line leads to values of p, q and r which are not all $\geqq 0$; hence this curve and the negative sign in the right-hand side do not show any real results and can therefore be neglected.

The value of C (16) has now been determined for different values of x and calculated by means of equations (14), (13) and (11). the corresponding values of z, y, p, q and r have been found. Table I gives the numerical values found for x, C, p, q and r.

Exact values for x, y, z, p, q and r have been determined for C = 10, 1 and 0.1

TABLE I.

Values for C, p, q and r as functions of x.

x	C	p	q	r
$\sqrt{3} = 1.732$	∞	0	0	$\sqrt{3} = 1.732$
1.8	81.167	0.012	0.015	1.785
1.9	31.638	0.031	0.039	1.861
1.95	23.940	0.041	0.051	1.899
2.0	19.124	0.052	0.063	1.937
2.1	13.443	0.073	0.088	2.013
2.2	10.213	0.096	0.113	2.087
2.4	6.696	0.145	0.166	2.234
2.6	4.838	0.193	0.217	2.383
2.8	3.702	0.252	0.275	2.525
3	2.939	0.318	0.338	2.662
3.5	1.846	0.5	0.5	3
4	1.274	0.709	0.673	3.327
4.5	0.934	0.960	0.862	3.638
5	0.714	1.216	1.050	3.950
6	0.455	1.841	1.436	4.535
7	0.314	2.575	1.910	5.091
8	0.229	3.421	2.382	5.619
9	0.174	4.374	2.876	6.125
10	0.136	5.431	3.391	6.610
11	0.109	6.588	3.922	7.078
12	0.089	7.846	4.472	7.529

TABLE II.

Values for x, p, q and r as functions of C.

C	x	p	q	r
∞	1.732051	0	0	1.732051
10	2.208596	0.097789	0.115195	2.093402
1	4.383533	0.889941	0.812619	3.570914
0.1	11.430635	7.118751	4.157819	7.272816

by the help of « regla falsi ». The results of these calculations are shown in Table II.

The deflections and angular deviations at the end of the beam have thus been obtained. It is now necessary to calculate the deflections, angular deviations, shearing forces and moments at the right of each of the points of elastic support.

To ascertain the deflection y_n and the angular deviations φ_n, it is assumed that the beam is considered at a point immediately to the left of the support n. The deflection y and angle of deflection φ_1 for the first support are

$$y_1 = y_n + (n-1)l\varphi_n + 2(n-1)^3\nu D_1 + 3(n-1)^2\frac{\nu}{l}M_1 - 2C\sum_{j=1}^{n-1}(n-j)^3 y_j - 3C\sum_{j=2}^{n-1}(j-1)(n-j)^2 y_j,$$

$$l\varphi_1 = l\varphi_n + 3(n-1)^2\nu D_1 + 6(n-1)\frac{\nu}{l}M_1 - 3C\sum_{j=1}^{n-1}(n-j)^2 y_j.$$

These equations give for y_n and $l\varphi_n$

$$\left.\begin{aligned} y_n &= y_1 - (n-1)l\varphi_1 + (n-1)^3\nu D_1 + 3(n-1)^2\frac{\nu}{l}M_1 - C\sum_{j=1}^{n-1}(n-j)^3 y_j,\\ l\varphi_n &= l\varphi_1 - 3(n-1)^2\nu D_1 - 6(n-1)\frac{\nu}{l}M_1 + 3C\sum_{j=1}^{n-1}(n-j)^2 y_j. \end{aligned}\right\} \begin{array}{c}(n = \\ 2,3\ldots\infty)\end{array} \quad (17)$$

By the help of these formulae y_2, $l\varphi_2$, y_3, $l\varphi_3$, etc., can be calculated. Thus :

$$\left.\begin{aligned} y_2 &= (1-C)y_1 - l\varphi_1 + \nu D_1 + 3\frac{\nu}{l}M_1,\\ y_3 &= (1-9C+C^2)y_1 - (2-C)l\varphi_1 + (8-C)\nu D_1 + (12-3C)\frac{\nu}{l}M_1,\\ y_4 &= (1-36C+17C^2-C^3)y_1 - (3-10C+C^2)l\varphi_1 + (27-16C\\ &\qquad + C^2)\nu D_1 + (27-36C+3C^2)\frac{\nu}{l}M_1,\\ y_5 &= (1-100C+135C^2-25C^3+C^4)y_1 - (4-46C+18C^2-C^3)l\varphi_1\\ &\qquad + (64-118C+24C^2-C^3)\nu D_1 + (48-204C+60C^2-3C^3)\frac{\nu}{l}M_1,\\ y_6 &= (1-225C+695C^2-298C^3+33C^4-C^5)y_1 - (5-146C+153C^2\\ &\qquad - 26C^3+C^4)l\varphi_1 + (125-560C+273C^2-32C^3+C^4)\nu D_1 + (75\\ &\qquad - 780C+573C^2-84C^3+3C^4)\frac{\nu}{l}M_1, \end{aligned}\right\} (18^a)$$

$$\left.\begin{aligned}
l\varphi_2 &= 3Cy_1 + l\varphi_1 - 3\nu D_1 - 6\frac{\nu}{l}M_1,\\
l\varphi_3 &= (15C - 3C^2)y_1 + (1 - 3C)l\varphi_1 - (12 - 3C)\nu D_1 - (12 - 9C)\frac{\nu}{l}M_1,\\
l\varphi_4 &= (42C - 39C^2 + 3C^3)y_1 + (1 - 18C + 3C^2)l\varphi_1 - (27 - 36C\\
&\qquad + 3C^2)\nu D_1 - (18 - 72C + 9C^2)\frac{\nu}{l}M_1,\\
l\varphi_5 &= (90C - 243C^2 + 63C^3 - 3C^4)y_1 + (1 - 60C + 42C^3 - 3C^3)l\varphi_1\\
&\qquad - (48 - 204C + 60C^2 - 3C^3)\nu D_1 - (24 - 306C + 144C^2\\
&\qquad - 9C^3)\frac{\nu}{l}M_1,\\
l\varphi_6 &= (165C - 1023C^2 + 636C^3 - 87C^4 + 3C^5)y_1 + (1 - 150C + 285C^2\\
&\qquad - 66C^3 + 3C^4)l\varphi_1 - (75 - 780C + 573C^2 - 84C^3 + 3C^4)\nu D_1\\
&\qquad - (30 - 936C + 1125C^2 - 216C^3 + 9C^4)\frac{\nu}{l}M_1.
\end{aligned}\right\} (18^b)$$

Introducing y_1 and $l\varphi_1$ from equation (1) into the preceeding equation, y_n and $l\varphi_n$ can be calculated for each value of C. Nevertheless in the case where $C = \infty$ (applicable to infinitely rigid supports), some extra investigations are necessary. It is not possible to give an infinite value to C in the formulae (18^a) and (18^b). Hence C_{yj} in the formula (17) is replaced by νK_j where K_j is the reaction in the jth. point of application. If $y_1 = 0$, and $y_n = 0$, the following equations result :

$$\left.\begin{aligned}
\nu\sum_{j=1}^{n-1}(n-j)^3K_j &= -(n-1)l\varphi_1 + (n-1)^3\nu D_1 + 3(n-1)^2\frac{\nu}{l}M_1,\\
l\varphi_n &= l\varphi_1 - 3(n-1)\nu D_1 - 6(n-1)\frac{\nu}{l}M_1 + 3\nu\sum_{j=1}^{n-1}(n-j)^2K_j.
\end{aligned}\right\} (n = 2, 3\ldots\infty) \quad . . . (19)$$

One can deduce

$$\left.\begin{aligned}
\nu K_1 &= -l\varphi_1 + \nu D_1 + 3\frac{\nu}{l}M_1,\\
\nu K_2 &= 6l\varphi_1 - 12\frac{\nu}{l}M_1,\\
\nu K_3 &= -24l\varphi_1 + 42\frac{\nu}{l}M_1,\\
\nu K_4 &= 90l\varphi_1 - 156\frac{\nu}{l}M_1,\\
\nu K_5 &= -336l\varphi_1 + 582\frac{\nu}{l}M_1,\\
\nu K_6 &= 1254l\varphi_1 - 2172\frac{\nu}{l}M_1,
\end{aligned}\right\} (20^a) \qquad
\left.\begin{aligned}
l\varphi_2 &= -2l\varphi_1 + 3\frac{\nu}{l}M_1,\\
l\varphi_3 &= 7l\varphi_1 - 12\frac{\nu}{l}M_1,\\
l\varphi_4 &= -26l\varphi_1 + 45\frac{\nu}{l}M_1,\\
l\varphi_5 &= 97l\varphi_1 - 168\frac{\nu}{l}M_1,\\
l\varphi_6 &= -362l\varphi_1 + 627\frac{\nu}{l}M_1.
\end{aligned}\right\} . (20^b)$$

The calculations for the shearing forces and their corresponding moments are easily made by considering a section immediately to the left of the nth. support and making use of equilibrium equations valid for the left-hand portion of the beam (see Fig. 1), i.e.:

$$D_n = D_1 - c \sum_{j=1}^{n-1} y_j, \quad M_n = (n-1)D_1 l + M_1 - c\,l \sum_{j=1}^{n-1} (n-j) y_j \quad . \quad . \quad (21)$$

For a value $C = \infty$ these equations may be written in the following form:

$$D_n = D_1 - \sum_{j=1}^{n-1} K_j, \quad M_n = (n-1)D_1 l + M_1 - l \sum_{j=1}^{n-1} (n-j) K_j. \quad . \quad . \quad (22)$$

If the values of ∞, 10, 1 and 0.1 are given to C the numerical values of the deflections are obtained by the use of equations (18^a), (18^b), (20^a), (20^b), (21) and (22) (for $C = \infty$: application forces) and in addition those for angular deflections, shearing forces and moments considered as far as the 6th. point of application, as shown in Table III.

3. Beams under a single load.

The important problem of a concentrated force P acting on a beam of infinite length resting on elastic supports will now be considered. As is shown in Fig. 4, the beam is considered as being cut into 3 sections. Only that portion of the beam on which the force acts will be considered, since the right and left hand portions correspond to the beam under consideration in Chapter 2.

The magnitudes at the right and left

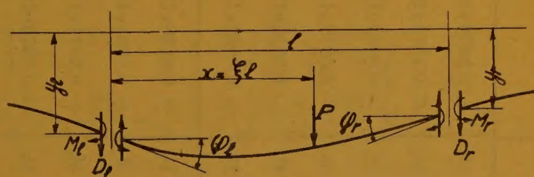

Fig. 4. — Beam of infinite length loaded with a concentrated load.

ends will be called l and r, and it will be assumed that the angles φ_l and φ_r are positive, as is shown in Fig. 4. According to formula (5) in Chapter 2, the following are obtained:

$$y_l = p\nu D_l + q\frac{\nu}{l} M_l, \; l\varphi_l = q\nu D_l + r\frac{\nu}{l} M_l, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$y_r = p\nu D_r + q\frac{\nu}{l} M_r, \; l\varphi_r = q\nu D_r + r\frac{\nu}{l} M_r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The distance between the left-hand end of the beam and the point of application of the force is called x and

$$\frac{x}{l} = \xi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

is obtained. By the help of the equations of equilibrium valid for this portion of the beam

$$D_l = P - D_r, \; M_l = \xi P l - D_r l + M_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

TABLE III.

Deflections, angular deflections, shearing forces and moments at points supports Nos. 1-6.

C	∞	10	1	0.1
y_1	$vK_1 = v D_1 + 1.2680 \frac{v}{l} M_1$	$+ 0.0978 vD_1 + 0.1152 \frac{v}{l} M_1$	$+ 0.8900 vD_1 + 0.8126 \frac{v}{l} M_1$	$+ 7.1188 vD_1 + 4.1578 \frac{v}{l} M_1$
y_2	$vK_2 = - 1.6077 \frac{v}{l} M_1$	$+ 0.0047 vD_1 - 0.1302 \frac{v}{l} M_1$	$+ 0.1874 vD_1 - 0.5709 \frac{v}{l} M_1$	$+ 3.2491 vD_1 - 0.5308 \frac{v}{l} M_1$
y_3	$vK_3 = + 0.4308 \frac{v}{l} M_1$	$- 0.0028 vD_1 + 0.0144 \frac{v}{l} M_1$	$- 0.0422 vD_1 - 0.2592 \frac{v}{l} M_1$	$+ 0.7832 vD_1 - 1.6610 \frac{v}{l} M_1$
y_4	$vK_4 = - 0.1154 \frac{v}{l} M_1$	$+ 0.0005 vD_1 + 0.0013 \frac{v}{l} M_1$	$- 0.0332 vD_1 - 0.0143 \frac{v}{l} M_1$	$- 0.2529 vD_1 - 1.2960 \frac{v}{l} M_1$
y_5	$vK_5 = + 0.0309 \frac{v}{l} M_1$	$- 0.0018 vD_1 - 0.0026 \frac{v}{l} M_1$	$- 0.0052 vD_1 + 0.0243 \frac{v}{l} M_1$	$- 0.4463 vD_1 - 0.6520 \frac{v}{l} M_1$
y_6	$vK_6 = - 0.0080 \frac{v}{l} M_1$	$+ 0.0013 vD_1 + 0.0003 \frac{v}{l} M_1$	$+ 0.0020 vD_1 + 0.0088 \frac{v}{l} M_1$	$- 0.3164 vD_1 - 0.1953 \frac{v}{l} M_1$
$l\varphi_1$	$+ 1.7321 \frac{v}{l} M_1$	$+ 0.1152 vD_1 + 2.0934 \frac{v}{l} M_1$	$+ 0.8126 vD_1 + 3.5709 \frac{v}{l} M_1$	$+ 4.1578 vD_1 + 7.2728 \frac{v}{l} M_1$
$l\varphi_2$	$- 0.4641 \frac{v}{l} M_1$	$+ 0.0489 vD_1 - 0.4507 \frac{v}{l} M_1$	$+ 0.4824 vD_1 + 0.0088 \frac{v}{l} M_1$	$+ 3.2934 vD_1 + 2.5202 \frac{v}{l} M_1$
$l\varphi_3$	$+ 0.1244 \frac{v}{l} M_1$	$- 0.0090 vD_1 + 0.0121 \frac{v}{l} M_1$	$+ 0.0541 vD_1 - 0.3904 \frac{v}{l} M_1$	$+ 1.6750 vD_1 + 0.1030 \frac{v}{l} M_1$
$l\varphi_4$	$- 0.0333 \frac{v}{l} M_1$	$- 0.0001 vD_1 + 0.0080 \frac{v}{l} M_1$	$- 0.0370 vD_1 - 0.1171 \frac{v}{l} M_1$	$+ 0.5123 vD_1 - 0.6363 \frac{v}{l} M_1$
$l\varphi_5$	$+ 0.0089 \frac{v}{l} M_1$	$+ 0.0066 vD_1 + 0.0066 \frac{v}{l} M_1$	$- 0.0169 vD_1 + 0.0082 \frac{v}{l} M_1$	$- 0.0357 vD_1 - 0.5848 \frac{v}{l} M_1$
$l\varphi_6$	$- 0.0025 \frac{v}{l} M_1$	$- 0.0287 vD_1 - 0.0313 \frac{v}{l} M_1$	$- 0.0008 vD_1 + 0.0153 \frac{v}{l} M_1$	$- 0.1788 vD_1 - 0.3268 \frac{v}{l} M_1$
vD_2	$- 1.2680 \frac{v}{l} M_1$	$+ 0.0221 vD_1 - 1.1520 \frac{v}{l} M_1$	$+ 0.1101 vD_1 - 0.8126 \frac{v}{l} M_1$	$+ 0.2881 vD_1 - 0.4158 \frac{v}{l} M_1$
vD_3	$+ 0.3396 \frac{v}{l} M_1$	$- 0.0249 vD_1 + 0.1496 \frac{v}{l} M_1$	$- 0.0773 vD_1 - 0.2417 \frac{v}{l} M_1$	$- 0.0368 vD_1 - 0.3627 \frac{v}{l} M_1$
vD_4	$- 0.0911 \frac{v}{l} M_1$	$+ 0.0027 vD_1 + 0.0060 \frac{v}{l} M_1$	$- 0.0351 vD_1 + 0.0175 \frac{v}{l} M_1$	$- 0.1151 vD_1 - 0.1966 \frac{v}{l} M_1$
vD_5	$+ 0.0243 \frac{v}{l} M_1$	$- 0.0020 vD_1 - 0.0069 \frac{v}{l} M_1$	$- 0.0020 vD_1 + 0.0318 \frac{v}{l} M_1$	$- 0.0898 vD_1 - 0.0670 \frac{v}{l} M_1$
vD_6	$- 0.0066 \frac{v}{l} M_1$	$+ 0.0160 vD_1 + 0.0191 \frac{v}{l} M_1$	$+ 0.0033 vD_1 + 0.0075 \frac{v}{l} M_1$	$- 0.0452 vD_1 - 0.0018 \frac{v}{l} M_1$
$\frac{v}{l} M_2$	$- 0.2680 \frac{v}{l} M_1$	$+ 0.0221 vD_1 - 0.1520 \frac{v}{l} M_1$	$+ 0.1101 vD_1 + 0.1874 \frac{v}{l} M_1$	$+ 0.2881 vD_1 + 0.5842 \frac{v}{l} M_1$
$\frac{v}{l} M_3$	$+ 0.0716 \frac{v}{l} M_1$	$- 0.0028 vD_1 - 0.0023 \frac{v}{l} M_1$	$+ 0.0327 vD_1 - 0.0543 \frac{v}{l} M_1$	$+ 0.2513 vD_1 + 0.2215 \frac{v}{l} M_1$
$\frac{v}{l} M_4$	$- 0.0195 \frac{v}{l} M_1$	$- 0.0001 vD_1 + 0.0037 \frac{v}{l} M_1$	$- 0.0024 vD_1 - 0.0368 \frac{v}{l} M_1$	$+ 0.1362 vD_1 + 0.0249 \frac{v}{l} M_1$
$\frac{v}{l} M_5$	$+ 0.0047 \frac{v}{l} M_1$	$- 0.0021 vD_1 - 0.0032 \frac{v}{l} M_1$	$- 0.0043 vD_1 - 0.0050 \frac{v}{l} M_1$	$+ 0.0464 vD_1 - 0.0421 \frac{v}{l} M_1$
$\frac{v}{l} M_6$	$- 0.0018 \frac{v}{l} M_1$	$+ 0.0139 vD_1 + 0.0159 \frac{v}{l} M_1$	$- 0.0010 vD_1 + 0.0026 \frac{v}{l} M_1$	$+ 0.0012 vD_1 - 0.0439 \frac{v}{l} M_1$

is obtained. Finally the equations of deformation can be deduced :

$$\left.\begin{aligned} y_r &= y_l + l\varphi_l + 2\xi^3\nu P + 3\xi^2(1-\xi)\nu P - 2\nu D_r + 3\nu\frac{M_r}{l}, \\ -\,l\varphi_r &= l\varphi_l + 3\xi\nu^2 P - 3\nu D_r + 6\frac{\nu}{l}M_2. \end{aligned}\right\} \quad . \; . \quad (5)$$

With the help of these equations, the formulae for D_r, M_r, y_r and φ_r, as expressed in P, ν, l, ξ and the magnitudes p, q and r are obtained. D_l and M_l of (4) are therefore inserted into equation (1) and the expression for y_l and $l\varphi_l$, are inserted into equation (5), as well as the expressions (2) for y^r and $l\varphi_r$. From this two equations for D_r and M_r can be deduced as follows :

$$\left.\begin{aligned} \frac{D_r}{P} &= \frac{2p + q + (2q + r)\xi + 3\xi^2 - 2\xi^3}{4p + 4q + r + 1}, \\ \frac{M_r}{Pl} &= \frac{2p + q + (2q + r)\xi + 3\xi^2 - 2\xi^3}{2(4p + 4q + r + 1)} - \frac{q + r\xi + 3\xi^2}{2(r + 3)} \end{aligned}\right\} \quad . \quad (6)$$

Substituting these expressions in (2)

$$\left.\begin{aligned} \frac{y_r}{\nu P} &= \frac{2p + q}{2}\,\frac{2p + q + (2q + r)\xi + 3\xi^2 - 2\xi^3}{4p + 4q + r + 1} - \frac{q}{2}\,\frac{q + r\xi + 3\xi^2}{r + 3}, \\ \frac{l\varphi_r}{\nu P} &= \frac{2q + r}{2}\,\frac{2p + q + (2q + r)\xi + 3\xi^2 - 2\xi^3}{4p + 4q + r + 1} - \frac{r}{2}\,\frac{q + r\xi + 3\xi^2}{r + 3}. \end{aligned}\right\} \quad . \quad (7)$$

is found.

In a similar manner, expressions can be found for D_l, M_l, y_l and $l\varphi_l$; the simpler method being to determine these magnitudes by the help of the formulae (6) and (7), replacing ξ by $l - \xi$.

The shearing force D_P to the left of force P and the bending moment M_P at this point, are

$$D_P = D_l,\; M_P = M_l - D_l\, l\xi, \quad . \quad (8)$$

so that the values

$$\left.\begin{aligned} \frac{y_P}{\nu P} &= \frac{y_l}{\nu P} + \xi\frac{l\varphi_l}{\nu P} + 3\xi^2\frac{M_l}{Pl} - \xi^3\frac{D_l}{P}, \\ \frac{l\varphi_P}{\nu P} &= -\frac{l\varphi_l}{\nu P} - 6\xi\frac{M_l}{Pl} + 3\xi^2\frac{D_l}{P} \end{aligned}\right\} \quad (9)$$

can be found for the deflection y and angular deflection $l\varphi_P$ of the beam shown in Fig. 5 at the right of the force P.

The corresponding magnitudes for the other points of support can be ascertained by means of the formula (6) and the data given in Table III for Chapter 2.

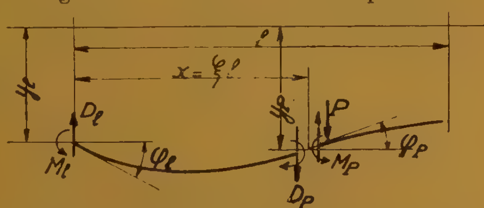

Fig. 5. — Portion of beam at left of P.

4. Consideration of two special cases.

It can easily be shown how the formulae obtained in the previous chapter are simplified for the special cases where the force is applied exactly on the point equidistant from two supports ($\xi = \frac{1}{2}$) and where the force is applied exactly on one of the supports ($\xi = 1$).

In the first case the following can be deduced from formulae (3, 6) and (3, 7)

$$\frac{D_r}{P} = \frac{1}{2}, \quad \frac{M_r}{Pl} = \frac{3 - 4q}{8(r + 3)}, \quad \ldots \ldots \quad (1)$$

$$\frac{y_r}{\nu P} = \frac{2p + q}{4} - \frac{q(4q + 2r + 3)}{8(r + 3)}, \quad \frac{l\varphi_r}{\nu P} = \frac{3(4q + r)}{8(r + 3)}. \quad \ldots \quad (2)$$

The magnitudes (3, 8) and (3, 9) at the point of application of the force are equal to

$$\frac{D_P}{P} = \frac{1}{2}, \quad \frac{M_P}{Pl} = -\frac{4q + 2r + 3}{8(r + 3)}, \quad \ldots \ldots \quad (3)$$

$$\frac{y_P}{\nu P} = \frac{p + q}{4} + \frac{16q + 4r + 3}{32(r + 3)}, \quad \frac{l\varphi_P}{\nu P} = 0. \quad \ldots \ldots \quad (4)$$

When $\xi = 1$, the formulae (3, 6) and (3, 7) become :

$$\frac{D_r}{P} = \frac{2p + 3q + r + 1}{4p + 4q + r + 1}, \quad \frac{M_r}{Pl} = \frac{2p + 3q + r + 1}{2(4p + 4q + r + 1)} - \frac{q + r + 3}{2(r + 3)}, \quad (5)$$

$$\frac{y_r}{\nu B} = \frac{(2p + q)(2p + 3q + r + 1)}{2(4p + 4q + r + 1)} - \frac{q(q + r + 3)}{2(r + 3)}, \quad \frac{l\varphi_r}{\nu P} = 0. \quad \ldots \quad (6)$$

The magnitudes at the point of application of the force are also expressed by equations (5) and (6).

It can easily be ascertained that the magnitudes at the point of application of the force corresponding to a value for ξ between $\frac{1}{2}$ and 1, are always found to be between the values of these magnitudes established when $\xi = \frac{1}{2}$ and $\xi = 1$.

Values for $\frac{\delta D_p}{\delta \xi}$, $\frac{\delta M_p}{\delta \xi}$, etc., are therefore calculated, and it can be shown that the deviations at intervals $\frac{1}{2} < \xi < 1$ never assume a zero value. It is not intended to prove this.

Deflections, angular deflections, shearing forces and bending moments at the point of application of the force and for the first six supports have been calculated and the results are incorporated in Tables IV[a] and IV[b].

5. The continuous support method.

It is now proposed to describe briefly the calculations necessary for a beam of infinite length supported on a continuous bed and on which a force P acts.

q indicates the magnitude of the vertical reaction (giving it the magnitude of the load acting on a unit of length) exercised by a small portion of the bed upon the beam which it supports when this produces a deflection y at the bed. It is assumed that the relation between q and y is shown by

$$q = \varkappa y \quad \ldots \ldots \quad (1)$$

If the flexibility c of the supports has been ascertained by the method explained in Chapters 2-4, and it is desired to use the continuous method, $\varkappa$ is selected in such a way that

$$\varkappa = \frac{c}{l}, \quad \ldots \ldots \quad (2)$$

where l equals the distance between two adjacent points of support.

TABLE IVa.

Deflections, angular deflections, shearing forces and bending moments for $\xi = \frac{1}{2}$.

C	∞	10	1	0.1
$y_P : vP$	+ 0.0656	+ 0.1343	+ 0.5697	+ 3.1397
$y_1 : vP$	0	+ 0.0561	+ 0.4411	+ 2.8697
$y_2 : vP$	0	− 0.0058	+ 0.0964	+ 1.7126
$y_3 : vP$	0	− 0.0005	− 0.0199	+ 0.6671
$y_4 : vP$	0	+ 0.0003	− 0.0165	+ 0.0885
$y_5 : vP$	0	− 0.0011	− 0.0027	− 0.1150
$y_6 : vP$	0	+ 0.0007	+ 0.0009	− 0.1258
$l\varphi_P : vP$	0	0	0	0
$l\varphi_1 : vP$	+ 0.1373	+ 0.1881	+ 0.3893	+ 0.8726
$l\varphi_2 : vP$	− 0.0368	− 0.0037	+ 0.2412	+ 1.2287
$l\varphi_3 : vP$	+ 0.0099	− 0.0037	+ 0.0289	+ 0.8204
$l\varphi_4 : vP$	− 0.0026	+ 0.0004	− 0.0180	+ 0.3617
$l\varphi_5 : vP$	+ 0.0007	+ 0.0037	− 0.0085	+ 0.0791
$l\varphi_6 : vP$	− 0.0002	− 0.0163	− 0.0005	− 0.0352
$D_P : P$	+ 0.5	+ 0.5	+ 0.5	+ 0.5
$D_1 : P$	+ 0.5	+ 0.5	+ 0.5	+ 0.5
$D_2 : P$	− 0.1005	− 0.0607	+ 0.0589	+ 0.2130
$D_3 : P$	+ 0.0269	− 0.0032	− 0.0375	+ 0.0418
$D_4 : P$	− 0.0072	+ 0.0017	− 0.0176	− 0.0249
$D_5 : P$	+ 0.0019	− 0.0014	− 0.0011	− 0.0338
$D_6 : P$	− 0.0005	+ 0.0092	+ 0.0016	− 0.0223
$M_P : Pl$	− 0.1708	− 0.1877	− 0.2548	− 0.4159
$M_1 : Pl$	+ 0.0792	+ 0.0623	− 0.0048	− 0.1659
$M_2 : Pl$	− 0.0212	+ 0.0016	+ 0.0541	+ 0.0472
$M_3 : Pl$	+ 0.0057	− 0.0016	+ 0.0166	+ 0.0889
$M_4 : Pl$	− 0.0015	+ 0.0001	− 0.0010	+ 0.0640
$M_5 : Pl$	+ 0.0004	− 0.0013	− 0.0021	+ 0.0302
$M_6 : Pl$	− 0.0001	+ 0.0079	− 0.0005	+ 0.0079

Differential equations $EI\frac{d^2y}{dx^2} = M$ and $\frac{d^2M}{dx^2} = -q$, can be applied. M is the moment produced in the beam, E the modulus of elasticity of the material of which the beam is composed and I its moment of inertia. Combining these two equations, the following differential equation is obtained

$$EIy'''' + \varkappa y = 0, \quad . \quad . \quad . \quad (3)$$

The introduction of the factor

$$\alpha = \sqrt[4]{\frac{\varkappa}{4EI}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

enables the above to be written

$$y'''' + 4\alpha^4 y = 0 \quad . \quad . \quad . \quad . \quad (5)$$

Co-ordinates are chosen in such a way that $x = 0$ to the right of P. In the case of (5) it is necessary to find a solution for y valid for $x > 0$ and a solution for y^* valid for $u < 0$, whereas for the limit-

TABLE IVb

Deflections, angular deflections, shearing forces and bending moments for $\xi = \frac{1}{2}$.

C	∞	10	1	0.1
$y_P : vP$	0	+ 0.0842	+ 0.5444	+ 3.1077
$y_1 : vP$	0	+ 0.0842	+ 0.5444	+ 3.1077
$y_2 : vP$	0	+ 0.0109	+ 0.2450	+ 2.3282
$y_3 : vP$	0	— 0.0033	+ 0.0130	+ 1.1356
$y_4 : vP$	0	+ 0.0004	— 0.0231	+ 0.3198
$y_5 : vP$	0	— 0.0015	— 0.0083	— 0.0482
$y_6 : vP$	0	+ 0.0012	— 0.0000	— 0.1342
$l\varphi_P : vP$	0	0	0	0
$l\varphi_1 : vP$	0	0	0	0
$l\varphi_2 : vP$	0	+ 0.0679	+ 0.3710	+ 1.2142
$l\varphi_3 : vP$	0	— 0.0089	+ 0.1103	+ 1.0592
$l\varphi_4 : vP$	0	— 0.0005	— 0.0080	+ 0.5741
$l\varphi_5 : vP$	0	+ 0.0057	— 0.0145	+ 0.1957
$l\varphi_6 : vP$	0	— 0.0248	— 0.0033	+ 0.0053
$D_P : P$	+ 1	+ 0.9212	+ 0.7722	+ 0.6554
$D_1 : P$	+ 1	+ 0.9212	+ 0.7722	+ 0.6554
$D_2 : P$	0	+ 0.0787	+ 0.2278	+ 0.3446
$D_3 : P$	0	— 0.0306	— 0.0172	+ 0.1118
$D_4 : P$	0	+ 0.0022	— 0.0302	— 0.0018
$D_5 : P$	0	— 0.0015	— 0.0071	— 0.0338
$D_6 : P$	0	+ 0.0137	+ 0.0012	— 0.0289
$M_P : Pl$	0	— 0.0507	— 0.1757	— 0.3747
$M_1 : Pl$	0	— 0.0507	— 0.1757	— 0.3747
$M_2 : Pl$	0	+ 0.0281	+ 0.0521	— 0.0301
$M_3 : Pl$	0	— 0.0025	+ 0.0348	+ 0.0817
$M_4 : Pl$	0	— 0.0003	+ 0.0046	+ 0.0890
$M_5 : Pl$	0	— 0.0018	— 0.0025	+ 0.0462
$M_6 : Pl$	0	+ 0.0120	— 0.0012	+ 0.0173

ing conditions the following are obtained :

$$\left.\begin{array}{l} y \text{ and its derivates} = 0 \text{ when } x = \infty, \\ y^* \text{ and its derivates} = 0 \text{ when } x = -\infty, \\ y = y^*, y' = y^{*\prime}, y'' = y^{*\prime\prime}, y''' = y^{*\prime\prime\prime} \\ \text{and } \dfrac{P}{EI} \text{ when } x = 0. \end{array}\right\} (6)$$

It can easily be verified that solutions for y and y^* of (5) which satisfy conditions (6) are :

$$\left.\begin{array}{l} y = \dfrac{P}{8\alpha^3 EI} e^{-\alpha x} (\cos \alpha x + \sin \alpha x), \\ y^* = \dfrac{P}{8\alpha^3 EI} e^{\alpha x} (\cos \alpha x - \sin \alpha x). \end{array}\right\} (7)$$

In order that this result may be compared easily with those found in previous chapters, a new variable is introduced

$$\xi = \frac{x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

as well as magnitudes ν and C (2, 2) and (2, 6) allows α to be put in the form of

$$\alpha = \frac{1}{l}\sqrt[4]{\frac{3}{2}C} \quad \ldots \ldots \quad (9)$$

and to indicate y in the form

$$\frac{y}{\nu P} = \frac{1}{2C}\sqrt[4]{\frac{3}{2}C}\, e^{-\xi\sqrt[4]{\frac{3}{2}C}}\left(\cos \xi\sqrt[4]{\frac{3}{2}C} + \sin\xi\sqrt[4]{\frac{3}{2}C}\right) \quad \ldots \quad (10)$$

and the bending moment in the beam $M = EI\,\frac{d^2y}{dx^2} =$

$$\frac{M}{Pl} = -\frac{1}{4}\sqrt[4]{\frac{2}{3C}}\, e^{-\xi\sqrt[4]{\frac{3}{2}C}}\left(\cos\xi\sqrt[4]{\frac{3}{2}C} - \sin\xi\sqrt[4]{\frac{3}{2}C}\right) \quad \ldots \quad (11)$$

Table V shows numerical values for y and M when C = 10, 1 and 0.1 and $\xi = 0, \frac{1}{2}, 1 \ldots 5$.

TABLE V.

Deflections and bending moments as functions of ξ.

C	10		1		0.1	
ξ	$y : \nu P$	$M : Pl$	$y : \nu P$	$M : Pl$	$y : \nu P$	$M : Pl$
0	+ 0.0984	— 0.1270	+ 0.5534	— 0.2259	+ 3.1116	— 0.4017
½	+ 0.0510	+ 0.0132	+ 0.4379	— 0.0423	+ 2.8677	— 0.1903
1	+ 0.0074	+ 0.0232	+ 0.2455	+ 0.0333	+ 2.3303	— 0.0495
1½	— 0.0041	+ 0.0078	+ 0.0954	+ 0.0466	+ 1.7064	+ 0.0329
2	— 0.0027	— 0.0000	+ 0.0122	+ 0.0346	+ 1.1360	+ 0.0725
2½	— 0.0006	— 0.0011	— 0.0196	+ 0.0184	+ 0.6662	+ 0.0835
3	+ 0.0001	— 0.0004	— 0.0232	+ 0.0066	+ 0.3196	+ 0.0775
3½	+ 0.0001	— 0.0000	— 0.0162	+ 0.0004	+ 0.0881	+ 0.0633
4	+ 0.0000	+ 0.0000	— 0.0082	— 0.0018	— 0.0484	+ 0.0467
4½	— 0.0000	+ 0.0000	— 0.0027	— 0.0019	— 0.1151	+ 0.0312
5	— 0.0000	+ 0.0000	+ 0.0001	— 0.0012	— 0.1341	+ 0.0184
5½	— 0.0000	+ 0.0000	+ 0.0010	— 0.0006	— 0.1257	+ 0.0090

6. Comparison of results obtained in Chapters 4 and 5.

In figures 6[a] and 6[b] the deflections and bending moments at the point of application of the force have been shown as functions of C for the two cases investigated in Chapter 4, where the force is applied at a point exactly mid-way between two supports and the other case where the force acts directly on one of the supports, as well as for the case considered in Chapter 5, where the beam was considered as supported by a continuous bed. As was stated at the end of Chapter 4, the deflection and the bending moment

at another point of application of the force are always found to lie between the limits shown in Figs. 6[a] and 6[b].

If the deflection values shown in Tables IV and V (where the 1st., 2nd., etc., points of support in Table IV[a] correspond respectively to values of $\xi = \frac{1}{2}$, $1\frac{1}{2}$, etc., of Table V and the 1st., 2nd., etc., points of support of Table IV[b] correspond to values $\xi = 0, 1$, etc., of Table V), are compared, variations in the deflection and the rigidity of the supports are found, and which are of the same order of magnitude as the deflections at the point of application of the force.

Bearing in mind that the maximum value of C normally found on the main lines of railways is equal to 2 (see Note (6) in Chapter 2), it is found from Fig. 6[a] that the deflections calculated by the method of continuous supports are situated between the maximum possible actual deflections and do not represent a difference greater than 5 % at the most.

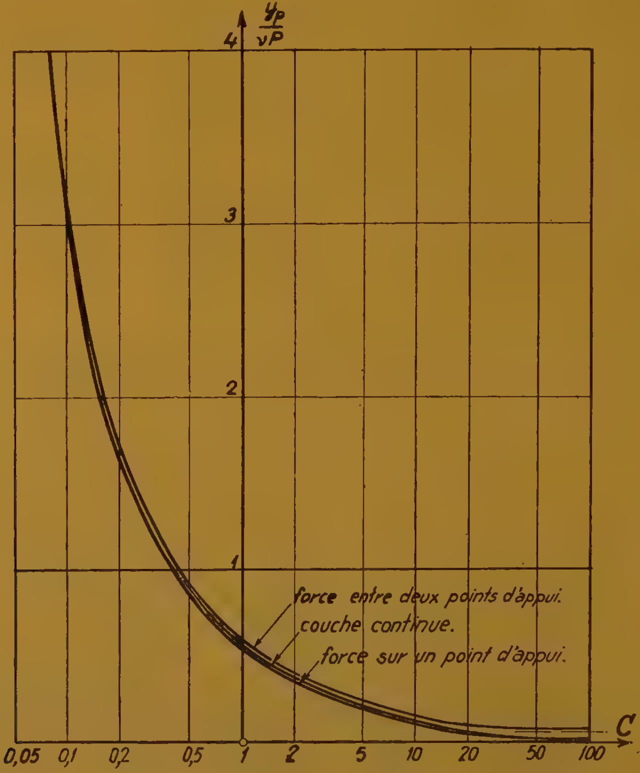

Fig. 6a. — Deflections at point of application of load as a function of C.

Explanation of French terms:

Force entre deux points d'appui = load applied between two supports. — Couche continue = continuous bed. — Force sur un point d'appui = load applied to one support.

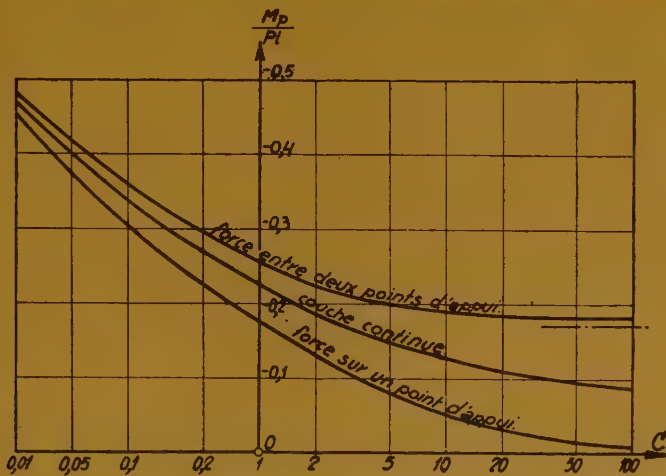

Fig. 6b. — Bending moments at point of application of load as a function of C.

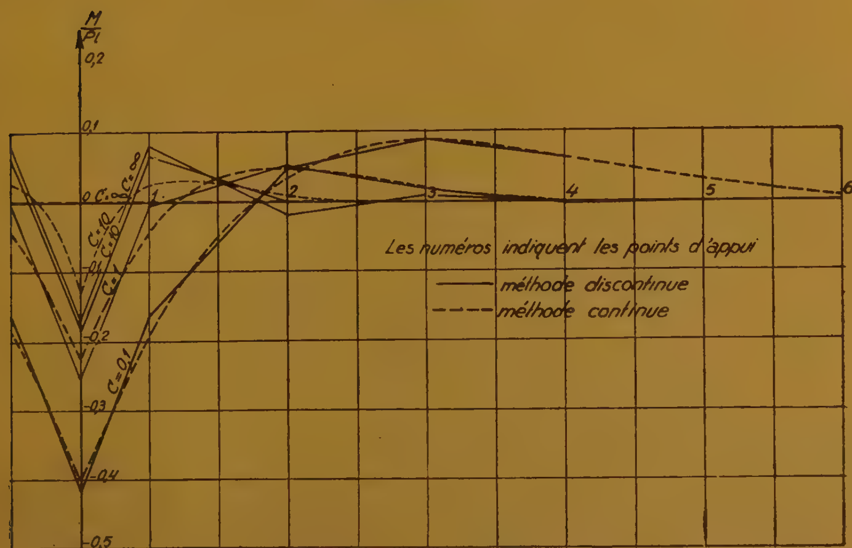

Fig. 7a. — Variation in bending moments along the length of the beam, load applied between two supports.

Explanation of French terms:

Les numéros indiquent les points d'appui = the numbers indicate the location of the supports. — Méthode discontinue = discontinuous method. — Méthode continue = continuous method.

Bending moments calculated by the method of continuous supports are also situated between the two moments calculated in Chapter 4, but the bending moments for calculated values of C often show greater divergences than do the deflections (see Fig. 6b). Figures 7a and 7b have been drawn to show the varia-

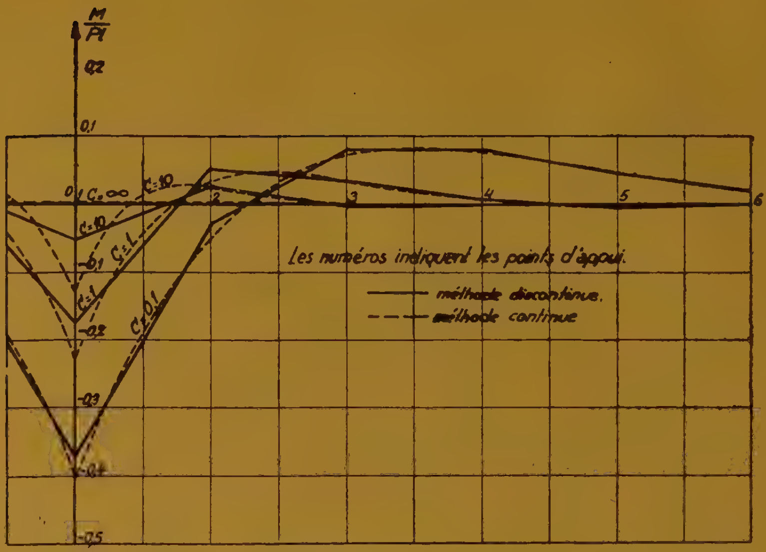

Fig. 7b. — Variation in bending moments along the length of the beam, load applied on one support.

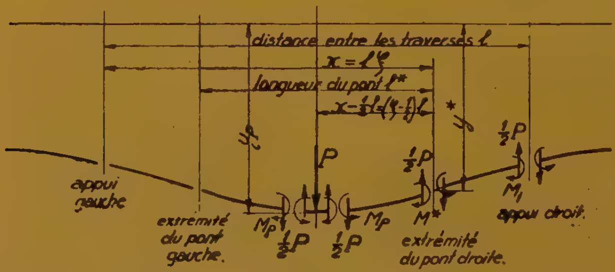

Fig. 8. — Plan of a portion of rail having a mercury contact treadle.

Explanation of French terms:

Distance entre les traverses = distance between sleepers. — Longeur du pont = length of bridge piece. — Appui gauche = left hand support. — Extrémité du pont gauche = left hand end of brigde piece. — Extrémité du pont droite = right hand end of bridge piece. — Appui droit = right hand support.

tions in the moments along the beam calculated by the method of continuous supports as well as by the method of separate supports.

7. The mercury contact treadle.

It is now proposed to investigate the effect of a mercury contact treadle, Siemens & Halske type, upon the supporting sleeper. It consists of a bridge piece or rigid section whose length is less than the distance between two sleepers and whose extremities are attached to the rail in such a manner that the centre of the bridge piece is exactly between two sleepers. A mercury contact is fixed in the

middle of the bridge piece. It is operated by the depression of the rail just above this point.

It is not intended to go into the constructional details of the treadle but only to investigate the depression, i.e. the displacement of the centre of the bridge piece with reference to that point on the beam situated immediately above when a force acts on that point. It is assumed that the bridge piece is absolutely rigid and that its extremities are attached to the rail by perfect joints.

Fig. 8 shows a portion of the beam. When a force P acts on the exact centre of the bridge piece, the deflection of the two ends equals y^*. The depression of the treadle under notice is equal to $y_P - y^*$.

For that portion of the beam situated between the point of application of the force P and right-hand end of the bridge piece, it is found that

$$y^* = y_P + \frac{M^* \ (\xi - \frac{1}{2})^2 l^2}{2EI} - \frac{P(\xi - \frac{1}{2})^3 l^3}{6EI} \quad (1)$$

With the assistance of one of the equations of equilibrium valid for the portion of the beam under consideration, the bending moment M^* can be expressed as a function of the bending moment M_P, i.e. :

$$M^* = \tfrac{1}{2}(\xi - \tfrac{1}{2})Pl + M_P \quad . \quad . \quad . \quad (2)$$

By introduction of ν (2, 4), the depression can be calculated from the formula (1) as

$$\frac{y_P - y^*}{\nu P} = -3(\xi - \tfrac{1}{2})^2 \frac{M_P}{Pl} - \tfrac{1}{3}(\xi - \tfrac{1}{2})^3 . \quad . \quad (3)$$

The bending moment M_P can be calculated by using the methods explained in Chapter 4.

The deformations shown by two treadles in use on the lines of the Netherlands Railway Company have been calculated for the values of C already stated. One of the treadles is of Siemens & Halske's new type, whose bridge piece has a length of 34 cm. (1′1³/₈″), whilst the other is of the same firm's old type, with a bridge piece 49.2 cm. (1′7²¹/₆₄″) long. The distance between two sleepers is supposed to be constant l = 60 cm. (1′11⁵/₈″). Hence in the new model $\xi - \frac{1}{2} = 0.2833$ and in the old model $\xi - \frac{1}{2} = 0.4100$. The amounts of depression as shown in Table VI were then calculated by the assistance of the data given in Chapter 4.

TABLE VI.

Depressions registered on a mercury contact treadle.

C	$\frac{y_P - y^*}{\nu P}$ new pattern	$\frac{y_P - y^*}{\nu P}$ old pattern
∞	0.0067	0.0517
10	0.0107	0.0602
1	0.0269	0.0940
0.1	0.0657	0.1753

Figure 9 shows these depressions plotted as functions of C. It will be noticed that the depression of the new type is considerably less than that of the old type, which permits one to arrive at the conclusion that the old type may be preferable to the new one in view of the importance of the bending of the treadle.

In order to arrive at some idea of the magnitude of these treadle depressions, they have been calculated for the numerical values used in the investigation referred to in Note (6) in Chapter 2. In this particular case $\nu = 6.10^{-6}$ cm./kgr.,

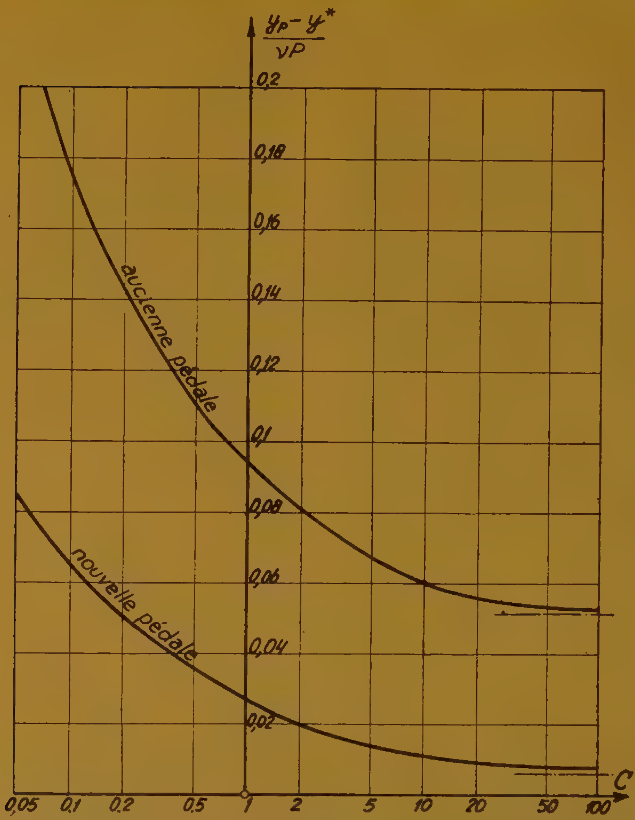

Fig. 9. — Amount of depression of a mercury contact treadle as a function of C.

Explanation of French terms:
Ancienne pedale = old type treadle. — Nouvelle pedale = new type treadle.

C = 0.18. If the applied force P equals 15 metric tons $\nu P = 0.09$ cm. The depression for a new type treadle can then be found according to the value of C:

$$y_P - y^* = 0.09 \times 0.053 = 0.00177 \text{ cm.}$$

and the depression for an old type treadle as

$$y_P - y^* = 0.09 \times 0.149 = 0.01341 \text{ cm.}$$

The depressions are thus of a very small order of magnitude.

8. Beam of finite length.

In view of the fact that these beams have but little bearing upon questions connected with railway rails, it is only intended to show in a succinct manner how the deformations and loads can be calculated for a beam of infinite length supported equidistantly (Fig. 10[a]).

It will first be assumed that the beam extends to infinity (Fig. 10[b]). In this case it is an easy matter to calculate the loads and deflections by substitution of

previous results obtained for various loads (which is permissible since the beam is of infinite length) as was shown in Chapter 3, as well as the moments M_l and M_d and the shearing forces D_l and D_r.

Then the deformations and loads are calculated for a beam which is loaded at the left-hand extremity by a moment M_l and a shearing force D_l (Fig. 10[c]). As can easily be seen, this can be done by calculating the moment M_k and the shearing force D_k at the right-hand side of the last support at the right, by applying the formulae (2, 21)in which the values y_j and φ_j are substituded by the values y_n and φ_n (2, 17) and $M_1 = M_l$, $D_1 = D_l$, $n = k$. The fact that the moment and shearing force at the right of the last support must equal zero leads to the conditions that $M_k = 0$ and $D_k = cy_k$. Two equations are thus obtained from which y_1 and φ_1 can be deduced. The other moments, shearing forces, deflections and angular deflections can then be ascertained by the assistance of the formulae (2, 17) and (2, 21).

In a similar manner, the loads and deformations of the beam produced by the moment M_r and the shearing force D_r can be determined (Fig. 10[c]).

Lastly the loads and deformations on the beam shown in Fig. 10[a] with the addition of the loads and deformations found in Figs. 10[b]-10[d] can be determined.

Fig. 10. — Beam of finite length.

It has not been verified whether this method is to be preferred in all cases to that given by KOITER, but it appears to present certain advantages over the calculations suggested by KOITER when it is desired to determine the lines of influence of a beam loaded with a single force. Further, this method can only be used for equidistantly placed supports, whilst KOITER's method can be used for supports placed at irregular distances.

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Superelevation and maximum speeds as functions of the radius of curves and of gradients,

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FOREWORD.

This article is an amplification of that on the same subject, published in the *International Railway Congress Bulletin* in December, 1933. It is divided into two parts :

The *first part* dealing with superelevation from the points of view of : 1) specific thrust or acceleration on the lines or on rolling stock; 2) the risk of vehicles overturning, and 3) the angle of the resultant of the forces on the inclined plane of the curves. A reconciliation of these points and a general conclusion with some explanations has also been considered advisable.

The *second part* deals with maximum speeds within various ranges, according to the ruling minimum curvature and maximum gradients. It concludes by advocating the necessity for providing a higher resistance to thrust for speeds above 90-105 km./h. (56-65 m.p.h.).

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SUPERELEVATION.

Chapter 1.

PART ONE.

SUPERELEVATION.

Chapter 1.

1. Purposes of superelevation.

Benefits and drawbacks.

The transversal inclination $\hat{\varphi}$ of the plane of horizontal curves is designed to apply the component mg sine φ (*) of the weight of the vehicles following the inclined plane, so as to counterbalance, wholly or in part, the effects of the centrifugal force $m\frac{v^2}{\rho}$ on the vehicles and on the *larger radius* or *outer rails* of the curve. It is obvious, a priori, that the cancelling-out of the centrifugal forces by the centripetal forces above can be effected only at a speed V_1, which was used for calculating the angle $\hat{\varphi}$ and which we call the *critical speed* of the curve of radius ρ and inclination $\hat{\varphi}$. For $V > V_1$, only a part of the centrifugal force is neutralised, the outer rails and the vehicles are *thrust towards the outside of the larger radius*. For $V < V_1$, there is an excess of centripetal force and the *smaller radius* rails and the vehicles are *thrust towards the centre of the curve*. The analysis which will be made in No. 2 hereafter will show this more clearly.

Maximum of the angle $\hat{\varphi}$. Inclination and corresponding superelevation.

For technical and economic reasons, the angle $\hat{\varphi}$ does not exceed the value φ_1, which is almost the same for any gauge

(*) (mg cos i) sine φ if the curve is inclined longitudinally; normally cos i $\approx$ 1.00.

of track. Most railways take $tg\varphi_1 = \frac{h}{e} = 0.100$; the P.L.M. allows 0.1066; the French Nord, owing to its excellent ballast goes to 0.120, and even to 0.133; the German Railways allow 0.080.

Corresponding to these inclines are the trigonometrical values of the angles $\hat{\varphi}_1$ and the corresponding degrees of superelevation h_1 below, in the case of normal gauge track, e = 1 500 mm. ($4'11^1/_{16}''$) :

$tg\varphi_1 = \frac{h_1}{e} = 0.08$	0.100	0.1066	0.120	0.1333...
$\hat{\varphi}_1 = 4^\circ\ 35'$	5° 43′	6° 5′	6° 51′	7° 36′
$\sin\varphi_1 = 0.0799$	0.0996	0.10597	0.11927	0.13226
$\cos\varphi_1 = 0.9968$	0.9950	0.99437	0.99286	0.99122
h_1 = 120 mm.	150 mm.	160 mm.	180 mm.	200 mm.

It will be noted : 1) that except on the French Nord, sineφ can always be replaced by $tg\varphi$ and cosφ by the unit 1; 2) that the maximum inclination for any gauge of track falls between 0.08 and 0.1066 — exceptionally 0.133 — and the corresponding degree of superelevation between 120 - 160 mm. ($4^{23}/_{32}$ - $6^5/_{16}$ inches), exceptionally 180 or 200 mm. ($7^3/_{32}$ or $7^7/_8$ inches). For any other track gauge e', the superelevation h' would be between $0.08e'$ and $0.1066e'$, h' and e' being expressed preferably in mm. The dimension most used is $h' = 0.100e'$, e' being reckoned as the distance between the axes of the rails.

In table No. 4 hereafter are shewn superelevation formulae for the most common track gauges and conditions stipulated.

2. Resultant of the centrifugal and centripetal forces ; direct and indirect action on vehicles and on curved track.

From the mechanical point of view, the resultant :

$$\Delta = \frac{mv^2}{\rho} - mg\frac{h}{e} \quad \ldots\ldots \quad (1)$$

of the centrifugal and centripetal forces is assumed to be transferred from the centre of gravity of the vehicles by means of the couple to the points of contact between the wheel tyres and the rails; it acts, according to its type directly or indirectly on the vehicles and on the curved rails — or has no effect — as follows :

First case.

The difference Δ *is nil.*

The speed v equalises the centrifugal and centripetal forces; their resultant Δ is nil and there is no action by either of the forces, neither on the vehicles nor on the rails.

Through trains and fast freight trains operate at about this *equalising speed v*, which will be given by the equation :

$$\frac{mv^2}{\rho} = m\frac{V^2}{12.96\rho} = mg\frac{h}{e} \quad . \ . \quad (2)$$

from which we obtain :

$$V_1 = 11.2756\sqrt{\rho\frac{h}{e}} \approx 11.3\sqrt{\rho\frac{h}{e}} \quad (3)$$

From the same equation (2) we get also the *theoretical formula*, or according to others the *mechanical formula*, No. 4, for superelevation :

$$h = e \frac{V_1^2}{127.14\rho} \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

h and e being expressed preferably in mm.

It should be noted that the equation No. 2 also employs the form 2′ :

$$g \frac{h}{e} = \frac{V_1^2}{12.96\rho} = 0.07716 \frac{V_1^2}{\rho} \quad . \quad . \qquad (2')$$

Second case.

The difference Δ is positive.

In this case the speed v causes the *centrifugal force to be greater than the centripetal force;* the outer rail is overloaded (see page 298) by comparison with the inner rail and is thrust with the vehicles towards the exterior of the large radius. If the overloading is large enough, that is to say if there is an insufficient degree of superelevation, the tractive effort E_e on the outer rail becomes greater than that E_i, on the inner rail; this sets up a pivoting movement of the locomotive towards the inner rail which is thus subjected to lateral thrust towards the centre of the curve, with additional wear on the lateral face.

Fast and express trains, by their speed, make the *difference Δ positive.* The incline $\frac{h}{e}$ is calculated from the theoretical formula No. 4, the speed V_1 being smaller than the maximum permissible speed through the curves so as to reduce the disadvantages of the superelevation to a slow train. In this respect the French P.O.-Midi and Est systems use in the theoretical formula :

first $V_1 = \frac{2}{3} V_{max} + 5$,

second $V_1 = \frac{2}{3} V_{max} + 20$ for $V_{max} \geqslant 60$;

the Italian Railways, $V_1 \approx \frac{3}{4} V_{max}$.

In the second case we would have :

$$\frac{mv^2}{\rho} = \frac{mV^2}{12.96\rho} > mg \frac{h}{e} \quad . \quad . \quad . \quad . \qquad (5)$$

or

$$V > 11.3 \sqrt{\rho \frac{h}{e}} = V_1 \quad . \quad . \quad . \quad . \qquad (6)$$

Dividing the difference Δ by m gives, in m./sec², the specific acceleration :

$$a_e = \frac{V^2}{12.96\rho} - g \frac{h}{e} \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

which applies directly on the vehicles and outer rails and possibly indirectly on the inner rails. Dividing a_e by g gives, in kgr./kgr. of the weight of the vehicles, the specific thrust :

$$p_e = a_e : g = 0.102\, a_e = \frac{V^2}{127.14\rho} - \frac{h}{e} \quad (8)$$

Third case.

The difference Δ is negative.

In this case the speed of the trains results in *the centrifugal force being less than the centripetal force;* the inner rails of the curve are overloaded and the thrust on these and on the vehicles is towards the centre of the curve. If the overloading is sufficiently heavy, that is to say if there is an excessive degree of superelevation, the lateral face of the

outer rails will be subject to the same conditions as the lateral face of the inner rails in the second case, in which case $E^i > E_e$.

The slower class of train, stopping trains, freight trains, etc., cause the difference Δ *to be negative.*

In this third case we have :

$$mg\,\frac{h}{e} > \frac{mv^2}{\rho} = \frac{mV^2}{12.96\rho} \quad \ldots . \quad (9)$$

or

$$\frac{h}{e} > \frac{V^2}{127.14\rho} \quad \ldots\ldots . \quad (10)$$

from which we get :

$$V < 11.3\sqrt{\rho\frac{h}{e}} = V_1 \quad . . \quad (11)$$

Dividing the difference — Δ by m gives, in m./sec² the specific acceleration :

$$a_i = g\,\frac{h}{e} - \frac{V^2}{12.96\rho} \quad \ldots\ldots . \quad (12)$$

which applies directly on the vehicles and inner rails, and possibly indirectly on the outer rails. Dividing a_i by g gives, in kgr./kgr. of the weight of the vehicles, the specific thrust :

$$p_i = a_i : g = 0.102a_i = \frac{h}{e} - \frac{V^2}{127.14\rho} \quad (13)$$

Conclusions. — 1) Only the critical speed V_1 cancels at the same time the specific acceleration or thrust on vehicles or rails in curves.

2) Above V_1, vehicles are subject to thrust, as are the larger radius rails, towards the exterior of the curve; the inner rails will also be thrust towards the centre of the curve if there is any considerable deficiency of superelevation.

3) Below V_1, vehicles are subject to thrust, as are the smaller radius rails, towards the centre of the curve; the outer rails will also be thrust towards the exterior of the curve if there is any considerable excess of superelevation.

Table 2 and the corresponding graph. 1 give a clear picture of the above analysis.

Chapter 2.

3. Superelevation in relation to specific acceleration or thrust on vehicles or curved track.

Formulae 7 and 12 above are for specific accelerations a_e and a_i; formulae 8 and 13, the specific thrusts p_e and p_i, and it has been shewn that at different speeds they apply, or otherwise, to vehicles and curved track.

Table 1 below has been compiled to show, for the French Est, the P.L.M. and the German Reichsbahn : 1) the *critical speeds* V_1 with the corresponding V_{max} speeds; 2) the maximum accelerations a_e and a_i, the latter being for the minimum speed of 20 km./h. (12 m.p.h.). It will be seen that the zone of acceleration a_i is much larger than that of a_e, and that the first is much larger than the second; thus we have :

1) *For the French Est :*

Average $a_e = 0.304$; average $a_i = 0.89 = 2.92a_e$.

2) *For the French P.L.M. :*

Average $a_e = 0.36$; average $a_i = 0.975 = 2.71a_e$.

3) *For the Reichsbahn :*

Average $a_e = 0.36$; average $a_i = 0.687 = 1.91a_e$.

To make clearer the details of Table 1, Table 2 and Graph. 1 have been drawn up to show the scope of accelerations a_e and a_i and their values as functions of the speeds indicated. For this purpose a curve of radius $\rho = 500$ m. (1640′5″) has been chosen for two cases — those of the French Est and the German Reichsbahn. We shall further discuss the question of the Hellenic State Railways, and their curves of 300 (984′3″) and 500 m. radius.

First case. — The French Est allows on the curves in question a V_{max} = 90 km./h. (56 m.p.h.) with $h = 150$ mm. ($5^{29}/_{32}$″), calculated from the theoretical superelevation formula, in which $V = V_1 = \frac{2}{3} V_{max} + 20 = 80$ km./h. (50 m.p.h.).

The arc *ab* of the graph. represents by its abscissae 90 and 80 km./h. the scope of the accelerations a_e (i.e. of trains whose speeds are between 80 and 90 km./h.) and by its ordinates, the values of the accelerations themselves, which become nil when $V_1 = 80$ km./h. The arc *bc* represents also by its extreme speeds 80 and 20 km./h. (12 m.p.h.) (*) the scope of the accelerations a_i (i.e. trains, whose speeds are between 80 and 20 km./h.) and by its ordinates, the values of the accelerations themselves, which become nil when $V_1 = 80$ km./h. It may also be noted from the graph. and numerical table that with trains of a speed less than 68 km./h. (42 m.p.h.) the accelerations a_i exceed the maximum initial accelerations a_e.

Second case. — The German railways allow on the curve in question (ρ = 500 m.) V_{max} = 85 km./h. (53 m.p.h.) with $h = 116$ mm. ($4^5/_8$″) calculated by the formula $h = 8 \frac{V^2_{max}}{\rho}$.

The arc *lm* of the graph. represents by its abscissae and ordinates the accelerations a_e which become nil at the characteristic speed $V_1 = 0.8249 \times 85 = 70.12$ km./h. (43.620 m.p.h.) and the arc *mn*, the accelerations a_i from a speed of 70.12 km./h. down to the minimum working speed, V_{min} = 20 km./h. It will also be seen that the zone of the accelerations a_e extends from 85 to 70.12 km./h. with relatively small ordinates, whilst the zone of the accelerations a_i goes from 70.12 km./h. to 20 km./h. with relatively large ordinates (below 51

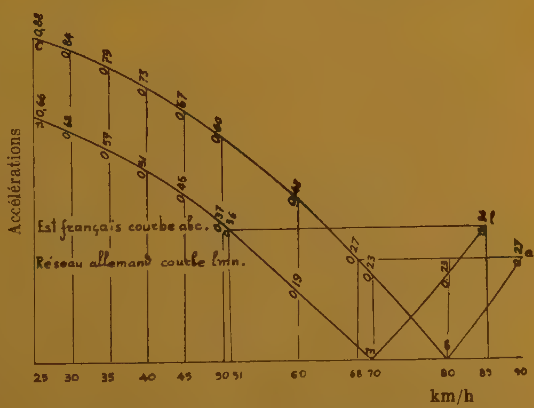
(*) See Table No. 2 for V = 20 km./h.

GRAPH. No. 1.

Scales : —

Speeds : ½ mm. to 2 km./h. (1.242 m.p.h.).

Accelerations : ½ mm. to 0.1 m./sec².



By inversion of 180° about the axis of the speeds (the abscissa) of the arcs *bc′* and *mn′* to the symmetrical positions *bc* and *mn* we get the position of the parabolic arcs *abc* and *lmn*, which are shewn algebraically in formula No. 7. In the above graph., the inversion of *bc′* and *mn′* to *bc* and *mn* has been done to give a positive value to the ordinates of a_i.

TABLE No. 1.

Speeds $V_{max.}$, V_1 and their maximum final specific accelerations for $V_{min.} = 20$ km./h.

System.	ρ =	*250*	*300*	*400*	*500*	*600*	*700*	*800*
French Est.	V_{max}	50	60	75	90	100	110	120
	V_1	»	»	70	80	87	93	100
	a_e	0.00	0.00	0.14	0.27	0.31	0.38	0.42
	a_i	0.00	0.00	0.87	0.92	0.92	0.91	0.93
French P.L.M.	V_{max}	65	73	85	95	105	115	125
	V_1	58	64	74	83	90	98	103
	a_e	0.26	0.32	0.35	0.35	0.37	0.41	0.46
	a_i	0.92	0.94	0.97	0.98	0.99	1.00	1.01
German Railways.	V_{max}	60	65	75	85	95	100	110
	V_1	49	54	62	70	78	82	91
	a_e	0.36	0.35	0.35	0.36	0.37	0.36	0.38
	a_i	0.63	0.63	0.66	0.69	0.74	0.70	0.75

TABLE No. 2.

Accelerations a_i, a_e for two particular cases.

French Est : $\rho = 500$ *m.* $h = 150$ *mm.*

For V = 20	25	30	35	40	45	50
a_i = 0.92	0.88	0.84	0.79	0.73	0.67	0.60

For V = 60	70	80	The accelerations a_i operate from 20 to 80 km./h.
a_i = 0.43	0.23	0.00	
For V = 80	85	90	The accelerations a_e operate from 80 to 90 km./h.
a_e = 0.00	0.13	0.27	

German Railways : $\rho = 500$ *m.* $h = 116$ *mm.*

For V = 20	25	30	35	40	45	50
a_i = 0.76	0.66	0.62	0.57	0.51	0.45	0.37

For V = 60	70		The accelerations a_i operate from 20 to 70 km./h.
a_i = 0.19	0.00		
For V = 70	80	85	The accelerations a_e operate from 70 to 85 km./h.
a_e = 0.00	0.23	0.36	

a_e and a_i in m./sec².

km./h. [31.69 m.p.h.], the accelerations a_i exceed the maximum initial accelerations a_e).

Third case. — On the *Hellenic State Railways*, superelevation is calculated by the formulae :

$$h_1 = \frac{40\,000}{\rho} \text{ for } \rho < 500 \text{ m.}$$

$$h_2 = \frac{50\,000}{\rho} \text{ for } \rho \geqslant 500 \text{ m.}$$

In these circumstances, characteristic speeds of 300 and 500 m. radius curves are easily calculated by formula No. 3; these are :

V_1 = 58 km./h. (36 m.p.h.) for $\rho < 500$ m.

and V_2 = 65 km./h. (40.38 m.p.h.) for $\rho \geqslant 500$ m.

By taking :

V_{max} = 65 km./h. for $\rho = 300$ m.

V_{max} = 85 km./h. for $\rho = 500$ m.

V_{min} = 20 km./h.

and using formulae 7 and 12, we get :

1) *For* $\rho = 300$ *m.* $\begin{cases} a'_e = 0.214 \\ a'_i = 0.768 \end{cases}$

2) *For* $\rho = 500$ *m.* $\begin{cases} a'_e = 0.461 \\ a'_i = 0.592 \end{cases}$

It will be noted that on the 300 m. radius curve, the zone of the accelerations a_e extends from 65 to 58 km./h. and that the initial maximum acceleration a'_e is less than the final maximum acceleration a'_i. On the 500 m. radius curve, the zone of the accelerations a_e extends from 85 to 65 km./h. with a smaller difference between the a'_e and a'_i; it may be noted that on the system in question, speeds do not exceed 70 to 80 km./h.

From the foregoing figures, and particularly from Graph. No. 1, we are led to conclude that superelevation, when designed for high speeds, is a disadvantage to slow trains, particularly freight trains, resulting in wear of the inner rail and involving, without any compensation, increased tractive effort in order to give a higher degree of protection to high speed trains than is really necessary.

4. a) Characteristic limits of specific accelerations a_e.

On the basis of the remarks on tangential acceleration and deceleration (see Table 3), the following characteristic limits for a_e have been proposed :

1) For any value of a_e *between 0 and 0.40 m./sec²* running is *smooth*, and the degree of smoothness increases as the value of a_e decreases.

3) For any value of a_e *between 0.40 and 0.60 m./sec²* inclusive, running is *reasonable*.

3) For any value of a_e *beyond 0.60 m./sec²* running becomes *rough*, and even *dangerous* if 0.60 m./sec² is too far exceeded.

It has already been remarked that the Italian Railways go up to a_e = 0.981 m./sec².

For comparative purposes, Table 3 includes the normal starting acceleration for steam and electric traction. For slow passenger trains the Italian Railways allow 0.50 m./sec².

It can be seen from Table 3 that only *tramways, suburban* and *railcar services* approach the characteristic limits beyond

0.40 m./sec². We believe, however, that the seated passenger is less susceptible to tangential acceleration than to normal acceleration.

The characteristic thrusts corresponding to the above limits are :

for a_e = 0.40 0.60 0.981 m./sec.²

p'_e = 40.80 61.2 100.6 kgr./t.

by using the formula $p_e = 0.102 a_e$.

These limits have been used in drawing up Table 4 for the track gauges normally used, these being measured between the axes of the rail heads. For track gauges other than those shewn in the table, the superelevation shewn on the third line of the table (normal gauge) should be multiplied by the ratio $e' : e$, e' being the gauge in question and e that of the normal track, both being expressed preferably in mm. In the table are noted the characteristics of the formula for each superelevation. Thus, in the case of the French Est, the theoretical superelevation (column *b*) would be, for trains of 90 km./h. (56 m.p.h.), h_1 = 191.16 mm.; it would not drop below :

1) h_2 = 161.16 mm. (column *c*) for trains of the same speed and for smooth running with a_e = 0.20 m./sec².

2) 130.16 mm. (column *d*) extreme limit for smooth running with a_e = 0.40 m./sec².

3) 99.16 mm. (column *e*) for the region of rough running and even dangerous running if $a_e > 0.60$ m./sec².

b) **Characteristic limits of a_i.**

Having adopted a maximum a'_e for a_e, we must consequently establish $\frac{h}{e}$, or

$$\frac{h}{e} = \frac{V^2_{max}}{127.14\rho} - \frac{a'_e}{g} \quad \ldots\ldots\ldots \quad (14)$$

From equation No. 12, its maximum is resolved :

$$a'_i = g\frac{h}{e} - \frac{V^2_{min}}{12.96\rho} \quad \ldots\ldots \quad (15)$$

which can no longer be fixed at will, since the two terms of the second item are determined, the first by equation No. 14, the second by the value of V_{min}, which goes down to 25 km./h. (15 m.p.h.) and even to 20 km./h. (12 m.p.h.) for goods trains and steep down gradients; thus, in the case of the French Est above, we should have :

$$a'_e = \frac{90^2}{12.96 \times 500} - 9.81 \times 0.100 = 0.269 \text{ m./sec}^2.$$

$$a'_i = 9.81 \times 0.100 - \frac{25^2}{12.96 \times 500} = 0.885 \text{ m./sec}^2.$$

In the article which appeared in the *International Railway Congress Bulletin* in December, 1933, the proposed maximum was a'_e = 0.30 m./sec², and another maximum was a'_e = 0.45 m./sec² or :

$$a'_e = \frac{2}{3} a'_i \text{ with the equation :}$$

$$a_e + a_i = \frac{V^2_{max} - V^2_{min}}{12.96\rho} \quad \ldots \quad (16)$$

which links together a_e and a_i.

From the foregoing analysis, a larger value of a'_e must be admitted, so that a'_i = 0.60 m./sec².

In these conditions we should have, with $V_{min} = 25$ km./h. :

$$V_{max} \leqslant 3.4 \sqrt{\rho + 55} \quad . \quad . \quad (a)$$

instead of

$$V_{max} = 3.1 \sqrt{\rho + 140} . \quad . \quad (b) \, (*)$$

The equations (a) and (b) give the same maximum speed of 69.6 km./h. (43.246 m.p.h.) for $\rho \approx 364$ m.; above this radius the equation (a) gives higher speeds than (b).

Formula (a) corresponds to an incline of $\frac{h}{e} = \frac{5}{\rho} + 0.06034$ and a superelevation of $h = \frac{7\,500}{\rho} + 90.51$ for normal gauge track.

(*) This formula was given in the article in the December, 1933, *Bulletin*.

Chapter 3.

5. Superelevation in relation to vehicles overturning.

It has been said above that specific acceleration is transferred from the centre of gravity of the weight of the vehicles to the points of contact between the rails and the wheel tyres by means of an overturning couple, to the *moment* of which is opposed the *moment* of the component $mg \cos \varphi \approx mg$ of the weight of the vehicles following the perpendicular to the inclined plane of the curves.

First case. — Overturning about the outer rails.

To avoid this, we should have :

$$\left(\frac{mv^2}{\rho} - mg\frac{h}{e}\right) \times \mathrm{H} < mg \cos \varphi \times \frac{e}{2} \approx mg \frac{e}{2} \quad . \quad . \quad (17)$$

TABLE No. 3.

Normal starting accelerations for steam and electric traction.

	Type of train :	*Steam.*	*Electric.*
a.	Goods	0.04 ÷ 0.05	0.09 ÷ 0.15
b.	Passengers	0.06 ÷ 0.07	0.13 ÷ 0.25
c.	Express, fast	0.08 ÷ 0.10	0.18 ÷ 0.25
d.	Railcars		0.30
e.	Tramways		0.50 ÷ 0.60
f.	Suburban		0.70 ÷ 1.00
Italian Railways :			
a.	Goods	0.05 ÷ 0.10	
b.	Passengers	0.10 ÷ 0.20	
d.	Railcars		÷ 1.10
e.	Tramways		0.40 ÷ 0.90
f.	Suburban		» »

a_e and a_i in m./sec².

or

$$\frac{V^2}{127.14\rho} < \frac{h}{e} + \frac{0.50e}{H} \quad . \quad . \quad (18)$$

H being the weight of the centre of gravity of the weight of the vehicles above the inclined plane of the curves.

It is easy to see that, even for very small degrees of superelevation, the inequality :

$$V < 11.3\sqrt{\left(\frac{h}{e} + \frac{0.50e}{H}\right) \times \rho} \quad . \quad . \quad . \quad (19)$$

allows higher speeds than those authorised in the curves; thus with $\rho = 500$ m., $H = 2$ m. and $h \approx 0$ we would have $V < 155$ km./h. (96 m.p.h.) to avoid overturning; with $H = 2.5$ m. we would have $V < 136$ km./h. (84 m.p.h.) whilst in a 500 m. radius curve the maximum permissible speed is 100 km./h. (62 m.p.h.).

Second case. — Overturning about the inner rails.

This is much more impracticable. In practice, the condition :

$$\frac{V^2}{127.14\rho} > \frac{h}{e} - \frac{0.50e}{H} \quad . \quad . \quad (20)$$

is always satisfied, even for the greatest value 0.100 of $\frac{h}{e}$, since the second item is negative at this value : it becomes positive if

$$\frac{h}{e} > \frac{0.50e}{H} = \frac{0.75}{H} = 0.375$$

for $H = 2$ m.; for normal gauge track, the superelevation to correspond to this

TABLE No. 4.

Formulae and limits for superelevation.

a	*b*	*c*	*d*	*e*
1 746	$13.74 \frac{V^2}{\rho}$	$13.74 \frac{V^2}{\rho} - 35$	$13.74 \frac{V^2}{\rho} - 71$	$13.74 \frac{V^2}{\rho} - 107$
1 670	13.14 »	13.14 » — 33	13.14 » — 68	13.14 » — 102
1 500	11.80 »	11.80 » — 30	11.80 » — 61	11.80 » — 92
1 060	8.34 »	8.34 » — 22	8.34 » — 43	8.34 » — 65
800	6.29 »	6.29 » — 16	6.29 » — 32	6.29 » — 49
650	5.11 »	5.11 » — 13	5.11 » — 26	5.11 » — 40

a. Width of track, between rail axes, in mm.
b. Theoretical or *mechanical* superelevation formula, in mm.
c. Minimum superelevation for average smooth running, $a_e = 0.20$.
d. Minimum superelevation below which smooth running ceases = $a_e = 0.40$.
e. Minimum superelevation below which running becomes dangerous = $a_e = 0.60$ and above.
Speeds are expressed in km./h.
Radii are expressed in m.
Superelevation is expressed in mm.

incline would be $h = 563$ mm. (1′10¹/₈″), which is quite unheard of.

In brief, overturning of vehicles outside the curve as a result of insufficient superelevation, or towards the inside of the curve as a result of excessive superelevation, is not to be considered a danger. Depression or spreading of rails and cross-winds on high viaducts or steep hill-sides, combined with high loading, are much more likely to cause overturning, particularly on narrow-gauge lines (e.g. 60 cm. [1′11⁵/₈″]). (See the excellent work on the same subject, « Traité élémentaire des Transports », p. 76, by J. CARLIER.)

Chapter 4.

6. Superelevation in relating to the variation $\underline{a}$ of the resultant of the weight of vehicles and the centrifugal forces on the inclined plane of curves.

It would be very desirable to keep the speed of trains in curved sections between limits compatible with a pre-determined superelevation, so that the resultant R of the weight P of the vehicles and the centrifugal force C passed through the axis of the inclined track. In practice, this is not easy to realise and the variation or deviation $\underline{a}$ from the resultant R is unavoidable, particularly with track over which runs trains of widely varying speeds.

The variation NM $= \underline{a}$ is given by formula No. 24 above, which is readily established as follows :

Let G be the centre of gravity of a vehicle, situated at height H above the inclined plane AB′ of the track.

GD $=$ P $= mg$ the weight of the vehicle.

GE $=$ C $= \dfrac{mv^2}{\rho}$ the centrifugal force due to the speed v.

GF $=$ R the resultant of the forces P and C.

$\hat{\varphi}$ the angle of transverse inclination of the track, or the angle between the direction GD of the weight P and the perpendicular GN over the inclined plane AB′.

$\hat{\Theta}$ the angle between the direction GD of the weight and the resultant GF of the forces P and C.

NM $= \underline{a}$ the length which may represent in its value the variation of the force R over the plane of the inclined track.

The following equations are evident from a study of Fig. 1 :

$$tg\,\Theta = \frac{C}{P} = \frac{mv^2}{\rho} : mg = \frac{V^2}{12.96\rho} \qquad (21)$$

$$tg\,\varphi = \frac{h}{e} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (22)$$

$$\underline{a} = \mathrm{H}tg(\Theta - \varphi) \approx \mathrm{H}(tg\,\Theta - tg\varphi) \quad . \qquad (23)$$

because of the low value of the angles $\hat{\Theta}$ and $\hat{\varphi}$.

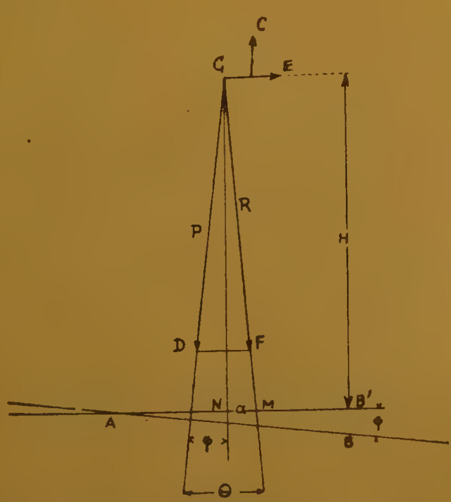


Figure 1.

Replacing $tg\Theta$ and $tg\varphi$ in the last equation by their values according to formulae 21 and 22, we have :

$$\underline{a} = \mathrm{H} \times \left[\frac{\mathrm{V}^2}{12.96\rho} - \frac{h}{e}\right] \quad . \quad . \quad . \quad . \quad (24)$$

whence we get

$$\mathrm{V} = 11.3\sqrt{\left(\frac{a}{\mathrm{H}} + \frac{h}{e}\right)\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

7. Coefficient of safety σ against vehicles overturning.

If the variation $\underline{a}$ becomes equal to the fraction $0.50e$ of the track width, the vehicles overturn.

Several systems limit train speeds so as to provide the condition :

$$\underline{a} = 0.50e \times \frac{1}{\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

σ being a figure between 3 and 6, and even up to 10; it is known as a *coefficient of safety against averturning for the speed given by formula 25.* Its value is obtained from the formula :

$$\sigma = \frac{0.50e}{\mathrm{H}} : \left[\frac{\mathrm{V}^2}{12.96g\rho} - \frac{h}{e}\right] \quad . \quad . \quad . \quad (27)$$

established with the aid of equations 24 and 26.

Reconciliation between the coefficients σ and the variations $\underline{a}$. — The following reconciliation between the position of the base M of the resultant R and the coefficient σ is easy to establish :

1) For the middle third of e $\sigma = 3$ $\underline{a} = 0.25$.

2) For the middle fifth of e $\sigma = 5$ $\underline{a} = 0.15$.

3) For the middle sixth of e $\sigma = 6$ $\underline{a} = 0.125$.

4) For the middle tenth of e $\sigma = 10$ $\underline{a} = 0.075$.

Abatement of rail load. — When the base M of the resultant R is on the limit of one of the above zones, the load on the wheels of the same axle is distributed between them according to the equations :

$$x = mg\left(0.5 + \frac{a}{e}\right) \text{for the exterior wheels,}$$

$$y = mg\left(0.5 - \frac{a}{e}\right) \text{for the interior wheels,}$$

and we get the following figures :

For $\underline{a}$ = 0.25 and σ = 3	0.15 5	0.125 6	0.075 10
x = 0.67	0.60	0.58	0.55
y = 0.33	0.40	0.42	0.45
y/x = 0.49	0.67	0.72	0.82

If point M of figure 1 falls to the left of point N over the same limits of the above zones, the off-loading is from the outer rails; the figures on the y line would be replaced by the figures against x and vice-versa.

The coefficients σ and the speed of safety and smooth running of trains in the U.S.A. — Most of the U.S.A. railways allow in curves a speed for V_{max} which is such that the resultant R passes through the middle third of the width of the track ($\underline{a}$ < 0.25 m. for normal gauge). The average coefficient σ_m which is thus obtained is equal to 6, and corresponds to $\underline{a}$ = 0.125 m. The so-called *safe speed* is that which corresponds to the coefficient σ = 3. No train is allowed to run at a speed in excess of the safe speed. Moreover, passenger trains are restricted to the *smooth speed*, that is, one which would demand a degree of superelevation in excess of 75 mm. ($2^{61}/_{64}''$), which in fact exists;

on the other hand, it is noted that passengers prefer the insufficient 75 mm. superelevation in comparison with that required by the running speed.

From these facts, and considering a curve of radius ρ = 500 m. (1640′5″) on the French Est and the German system, which requires a superelevation of 150 mm. ($5^{29}/_{32}$″) for $V_{max} = 90$ km./h. (56 m.p.h.) and of 116 mm. ($4^{27}/_{64}$″) for $V_{max} = 85$ km./h. (53 m.p.h.), one can have a minimum superelevation of 75 mm. ($2^{51}/_{64}$″) in the first case and of 41 mm. ($1^{39}/_{64}$″) in the second.

Trains could still run through a curve of 1 536 m. (5 039′$4^1/_4$″) radius, which requires in the case of the French Est for $V_{max} = 120$ km./h. (74.564 m.p.h.) a superelevation of 110.63 mm. ($4^{23}/_{64}$″) with h = 35.63 mm. ($1^6/_{16}$″) only; in the case of the German system no superelevation would be necessary; for this curve we should have $h = 8 \times \frac{V^2}{\rho} =$ 75 mm., the lack of which is preferred by the passengers.

8. Relation between the accelerations a_ρ, a_i, the variation $\underline{a}$ and the coefficients of safety σ.

Replacing in formula 24 the brackets by their equivalents $a_e : g = 0.102a^e$ (according to formula 7) we should have :

$$\underline{a} = Ha_e : g = H.0.102a_e \quad . \quad . \quad . \quad (28)$$

$$\text{or } \sigma = 0.50e : H.0\ 102a_e = 0.75 : H.0.102a_e$$
$$= 7.353 : Ha_e. \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

in the case of the normal gauge track.

With the help of equation No. 29, Table No. 5 has been drawn up for the values of H of 1.25 to 3 m. (4′$1^7/_{32}$″ to 9′$10^1/_8$″) and for the coefficients of safety from 3 to 10.

The values of the accelerations a and a_i of this Table shewn above the first stepped line indicate that we are *above the zone of safe working*. Between the two stepped lines is *the safe working zone* and only the values shewn below the second stepped line correspond to the *smooth running*. From this we know that those which use coefficients of safety, even of σ = 10, allow accelerations much larger than those characterised by the limits a_e = 0.40 m./sec² for smooth running and a_e = 0.60 m./sec² for safe running. All things equal, however, wear of rails and tyres will be greater than in the case of the above limits; the smoothness of running will also be reduced. Possibly the superior quality of American steel and the large axle loads, by comparison with other countries, justify the adoption of such large accelerations on their systems.

Note. — If, in equation 19, we know only one term of the expression $\frac{0.50e}{H}$ say $\frac{1}{\sigma} \times \frac{0.50e}{H}$ for a higher degree of safety, we take the formula No. 26, following the equation $a = \frac{0.50e}{\sigma}$. Therefore the questions raised in Nos. 5, 6 and 7 result in one purpose, the danger of overturning of the vehicles.

We will repeat here that the low values of the coefficients of safety is the uncertainty of the values to be attributed to H; they can be calculated for locomotives but not for the various types of other vehicles and their loads.

GENERAL CONCLUSIONS.

a) Except on large-radius lay-out (ρ = 800 m. [2 624′8″]), the superelevation provided, and calculated for high speeds, is too great for slow trains.

TABLE No. 5.

Accelerations a_e and a_i as functions of the co-efficients of safety σ and heights H.

σ \ H	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
	a_e or a_i							
3	1.96	1.63	1.40	1.25	1.08	0.98	0.89	0.82
4.08	1.43	1.20	1.03	0.90	0.80	0.72	0.66	0.60
4.41	1.33	1.11	0.95	0.83	0.74	0.67	0.60	0.56
4.9	1.20	1.00	0.86	0.75	0.67	0.60	0.55	0.50
5.5	1.07	0.90	0.76	0.67	0.60	0.53	0.49	0.45
6.13	0.96	0.80	0.69	0.60	0.53	0.48	0.44	0.40
7.0	0.84	0.70	0.60	0.53	0.47	0.42	0.38	0.35
8.17	0.72	0.60	0.51	0.45	0.40	0.36	0.33	0.30
9.8	0.60	0.50	0.43	0.38	0.33	0.30	0.28	0.25

Formulae : $\dfrac{a_e}{g} = \dfrac{V^2}{127.14\rho} - \dfrac{h}{e}$

See fig. 1.
$$\begin{cases} a = H.\dfrac{a_e}{g} = H.\,0.102\,a_e \\ \sigma = \dfrac{0.5e}{a} = \dfrac{7.353}{H.a_e} \\ a_e = \dfrac{7.353}{H.\sigma} \end{cases}$$

a_e and a_i in m./sec².

Tables 1 and 2, also graph. No. 1, leave no doubt on this point. The calculation of the critical speed V_1 by the use of formula 3, will quickly show the extent of the active zones of the accelerations a_e and a_i and the maximum values of the accelerations. Greater demands than are necessary are made on inner rails, wheel tyres and smooth running of slow trains to give an excessive degree of protection to fast services.

b) The danger of vehicles overturning, due to excessive centrifugal or centripetal force, does not exist, even at speeds much higher than those normally used in curves. The danger could arise, particularly on narrow gauge lines (600 mm. [2′] for example) if the force of a very strong cross-wind were added.

c) The coefficient of safety σ should start at 4 and rise to 10, instead of 3 to 6 as normally used. In all cases, they correspond to large accelerations, particularly in the case of low centres of gravity (flat wagons with heavy and low loading, rails, etc.). Their small values are due to the uncertainty of the values to be given to H, values which vary not only with the type of vehicle but also according to the conditions of loading.

Notes. — *a*) Inclination, in relation to the vertical axis over curves should be the same as on the straight so that the tyre treads fit the rails properly and the latter are not loaded obliquely. This, however, is not the case; after superelevation has been effected, the outer rails have an inclination towards the centre of

the track, corresponding to an angle $2°52' + \varphi$ of which the first term corresponds to a conicity of $\frac{1}{20}$; the inner rails would have, on the other hand, an inclination corresponding to the angle $2°52' - \varphi$ a negative incline $\varphi > 2°52'$, i.e. for a superelevation of normal gauge track greater than 75 mm. ($2^{61}/_{64}''$).

This practice involves the disadvantages : the forces of the weights change neither sense nor direction by the inclination φ of the plane of the curves and of the axles of the vehicles; it should be so with an inclination of the vertical axis of the rails. In any case, in the extreme position, towards the centre of the curve, the vertical axis of the inner rail should be vertical; it is not so, however, since $h > 75$ mm. It would be desirable that the resultant of the forces acting on the head of the rail should not apply outside the middle tenth of the width of their base $\left(\frac{1}{8.9}\right.$ for the standard French rail of 46 kgr./m. (92 lbs/yd.). Current practice in this respect is allied to the practice of sleeper lining; the same transversal inclination of $\frac{1}{20}$ is applied to sleepers on curves, which is given to the rail shoes on straight alignments.

b) In establishing the difference Δ, we have not taken account of the points raised in the preceding remark concerning the inclined running surface of the rails, nor of the degree of friction ($mg \cos\varphi$) $\times f$, which is the normal component on the inclined plane of the curve of the weight mg of the vehicles; these have been omitted, principally because of the small amount of play between the tyre flanges and the interior surfaces of the heads of the rails : the inclination so calculated is slightly increased (see VANDERRYDT and MINSART, « Cours d'Exploitation des chemins de fer », and the work by COUCHE [Vol. 1, p. 143]).

With regard to indirect effects of specific acceleration on the rails of curves, see « Cours de chemins de fer », by Professors VICAIRE and MAISON, p. 418, which should also be consulted in regard to superelevation and parabolic transition curves in plan and in profile.

c) We have discussed in this article the value h of superelevation or of the corresponding incline $\frac{h}{e}$, without going into the important question of the necessary transition curves. The reason for this is that the subject has been dealt with in mathematical sequence but without definite solutions; in fact, the mathematical continuity both as regards the radius of curvature and the inclination, of the position and of transition curves of the inclined plane used to arrive at the degree of superelevation, comprises only approximate solutions, whose values vary according to the compiler. Professor DESCUBES has dealt with this subject in his « Cours lithographié de l'Ecole Nationale des Ponts et Chaussées » (Session 1921, Annexe No. 2). Professors VICAIRE and MAISON, VANDERRYDT and MINSART and D'OCAGNE have also dealt with the same question (see part 2 of this article for works to be consulted).

(To be continued.)

Athens, 6th August, 1947.

[621 .33 (.44)]

The development of electrification of the French Railways.

The choice of system: 3 000v. D. C. 15 000v. Single phase, or 1 500v. D. C.,

by André GACHE.

(Revue de l'Association française des Amis des Chemins de fer, May-August 1946.)

It may occasion some surprise to touch on the topic of choice of voltage and system for the electrification of the French railways, firstly because the question has already been decided by the Ministerial Order dated 29th August 1920, which completely excluded the use of single phase current and recommended direct current at 1 500 volts, while permitting the use in special circumstances of 750 V. of 3 000 V. The final decision to employ direct current at 1 500 V. was the outcome of an exhaustive investigation in the course of which inquiries had been made of electrified systems both in Europe and in America. The programme prepared at the end of the 1914-1918 war covered a minimum of 6 000 km. (3 728 miles) (it is well known that only half this programme was completed, and even now the lines which have since been electrified do not correspond with those which were decided upon in 1919); it called for a system of traction which, if not well tried, was at least susceptible of being carried out with the maximum of ease and the minimum of time. There is no doubt that under these conditions only the 1 500 V. D.C. system could have proved satisfactory to the French railways, and for our part we consider that the choice on which the Ministerial Order finally settled was entirely justified. Twenty-five years have since passed. All countries have had recourse to a greater or lesser extent to electricity for rail traction; and after the elimination of the three phase system — self-condemned by the complexity of its contact lines — the only systems which have survived are single phase 11 000 or 15 000 V. at 25 or $16^2/_3$

$B'_0B'_0B'_0$ class E.626 locomotive, Italian Railways. Continuous output 2 700 metric H.P. Weight 96 tonnes (94.483 Engl. t.) 3 000 V. D.C.

cycles, and direct current at 1 500 or 3 000 V.

It will be recalled that the Pennsylvania Railroad and the New York New Haven and Hartford of U.S.A. have adopted single phase 11 000 V. - 15 cycles; Switzerland, Germany, Sweden and Norway have chosen single phase 15 000 V. - $16^2/_3$ cycles. Direct current at 3 000 V. has been selected by the Chicago-Milwaukee (U.S.A.), Italy, Belgium, the Morroccan Railways and Brazil; while 1 500 V. D.C. has developed mainly in France, the Dutch Indies, Holland, Spain and Japan.

One question immediately presents itself. If our choice of 1 500 V. was entirely reasonable in 1920, why were we led to reconsider the problem immediately before the Paris-Lyons electrification? Did this not prove without doubt that other traction systems had reached a pitch of technical perfection equal to that of our own 1 500 V.? Had many of the drawbacks normally associated with these other systems been overcome? It is on these points that we shall endeavour to shed some light by making reference to recent developments both in France and abroad.

The second cause for surprise which may well become evident in the course of reading this article is that the subject has already been covered in the « Revue Générale des Chemins de Fer », and in previous articles on electric traction we have felt it to be unnecessary to dwell on a question which had already formed the subject of an article in another journal. However, in this case the circumstances are not the same, since the subject is susceptible of consideration from different aspects; and this method of approach may lead to emphasis being placed on certain points which might otherwise escape attention. Finally, the particulars on which we have relied for our information are not the same as

Dessau-Bitterfeld line (Germany). Train of 1910. 2'B1 locomotive No. 10501. Siemens class ES1. Single motor. Vertical rod drive. Single phase 15 000 V.

those on which the article in the R.G.C.F. was based. This circumstance may be seen to have one advantage, which is moreover of more than ordinary interest, since in the end, though for different reasons, we reach exactly the same conclusion as the R.G.C.F., i.e. that we shall be well advised to continue to use the 1 500 volt system on our electrification schemes.

A. The 3 000 volt D.C. system.

With the use of a 3 000 volt D.C. system, the derivation of power from a national distribution network presents no difficulty. Up to a few years ago, however, this could only be effected by means of motor generator sets, since electrical science knew no practical method of producing 50 cycle rotary convertors for such a high voltage (there are of course in France a few 1 500 V. rotary convertors, but the majority of substations are equipped with two 750 V. rotaries operating in series). These motor generator sets are extremely reliable in operation, but they have high no-load losses (about 10 %), and have the disadvantage of a low efficiency, which with a poor load-factor falls to some 60 %.

The mercury vapour rectifier produced during the last ten years now offers a complete solution to the problem of linking a 3 000 V. D.C. traction system to the 3 phase 50 cycle national network. It should however be noted that, with rectifiers, the use of regenerative braking is very difficult, and leads to complications which, it cannot be denied, make it an uneconomical proposition.

With 3 000 V., the substations may have a spacing very nearly four times that which would be selected for 1 500 V., other things being equal, and assuming the same section of catenary and feeders. As has been advocated by Mr. PARODI for many years, it would be advisable to adopt a system of distributed substation capacity, for this ultimately leads to a lower installed capacity combined with a more constant line voltage (with the present Italian system of concentrated capacity, voltage drops of the order of 300 V., or 10 %, are frequently observed). With this arrangement, and in view of the very dense traffic to be provided for on the Paris-Lyons line, it would be necessary to adopt an average substation spacing of some 30 km. (18 miles).

However, it is very unlikely that the cross-section of catenary used for 1 500 V. would be retained for 3 000 V. A closer spacing would therefore have to be adopted, and ultimately the saving which might be expected to be obtained on the cost of the fixed equipment would in any case cease to be a governing factor in the choice of voltage. — Does this mean, then, that the decision would have to rest on better characteristics possessed by the rolling stock ?

Such is not considered to be the case. Locomotives and motor coaches for 3 000 V. D.C. cannot be equipped with motors wound for 3 000 V. : though it is actually possible to provide small outputs of some 10 H.P. at this voltage, electrical development has not yet reached the stage of producing motors of from 250-1 000 H.P. « Parallel » motor connection cannot therefore be used on 3 000 V. Moreover, to obtain a sufficient degree of reliability, it is necessary in practice to use twin motors with armatures wound for 1 500 V. and mechanically coupled; all the high-speed Italian locomotives are so equipped. Although this arrangement enables overvoltages due to wheel-skid to be avoided, it has the grave disadvantage of producing difficulties in the locomotive's mechanical design. It should also be noted that, other things being equal, a 1 500 V. motor intended for operation in series

on 3 000 V. must be insulated for this voltage, and so is heavier than a standard 1 500 V. motor. For motor coaches having to operate at speeds above 90 km./h. (56 m.p.h.) nose suspension must therefore be abandoned and recourse must be had to quill drives (e.g. on the Brussels-Antwerp trains, and on Italian motor coaches). Finally, even with field shunting it is still not possible to obtain a range of economical running notches to compensate for the loss of one method of grouping.

Certainly, so far as we know, no 1 500 V. compensated motors intended for operation two in series on 3 000 V. have been constructed; yet compensation alone enables high percentages (more than 75 %) of field shunting to be employed in conjunction with good commutation. It is true that, basically, there are no insurmountable difficulties in compensating 3 000 V. locomotive motors; but, even if this were done, there would always be a smaller degree of flexibility (owing to the loss of the parallel connection) than with the 1 500 V. locomotive.

We are aware that the opinion which we have just expressed on the subject of 3 000 V. stock is not shared by a number of engineers. With regard to the more particular question of using 1 500 V. motors connected two in series on 3 000 V., before giving this attention would it not be of use to give an entirely impartial consideration to our own 1 500 V. motors ?

We see at once that apart from the 75 motor coaches designed by the Paris

Photo Bègue.

Present appearance of the S.N.C.F.'s last single phase locomotive, a survivor of the six built for trial in 1910. Perpignan-Villefranche line. Single phase 12 000 V. — 16.66 cycles.

Metropolitan for the Sceaux line, we do not possess in France any motor coach equipped with 1 500 V. motors; all have motors wound for 675 or 750 V. connected two in series across 1 500 V. On 3 000 V. therefore these coaches would have but a single motor group — 4 motors permanently in series. The same would also apply to the Sceaux line motor coaches equipped with two motors only. In this connection, the lightweight coach motor wound for 1 500 V. has yet to be produced (the 235 H.P. 1 500 V. Sceaux line motor actually weighs 2 919 kgr. (6 435 lbs.) whereas the 235 H.P. 675 V. Budd unit motor weighs only 1 756 kgr. (3 872 lbs.).

With regard to express locomotives, only the « 500 » class is equipped with motors having 1 350 V. on the commutators; these have an excellent performance by virtue of their compensating windings. It will be remembered that it was locomotive E.502, at the start of the 1 500 V. electrification of the Orleans Company's lines, which was equipped with the first compensated motors — at the instigation of Mr. PARODI, be it noted. The performance of this type of motor has amply justified its production. Every other class has twin motors wound for 450, 675, or 750 V. per commutator (South Western Region locomotives 701-702, 704 to 720, 4801-4824; and South Eastern Region Nos. 242A.E.1, 242B.E.1, 262A.E.1-4); or triple motors wound for 675 V. per commutator (South Western Nos. E.703, 2D2.5302 to 5306).

Photo B.B.C.

St. Gothard express at Airolo (1921). Brown-Boveri 1BB1 type locomotive, 12302-12342 series. 15 000 V. single phase.

We have no twin motor wound for 1 500 V. per commutator. Although there are no very great difficulties to be anticipated in producing one, we should, however, have to gain our own experience. For high speed locomotives at 3 000 V., their production may provide some surprises for us, however simple it may appear.

The most interesting case is undoubtedly that of the mixed traffic locomotives, especially those of $B'_oB'_o$ type. We know that, apart from the 30 Western locomotives BB011 to 040, the six driving axle South-Eastern locomotives and the South Western Metadyne loco-

motives, all the mixed traffic stock is equipped with single 1 500 V. or 1 350 V. motors with 40 %-50 % maximum field weakening. It cannot be said that these motors have perfectly black commutation at all outputs. Although of generally robust construction, whatever type they may be, and even among those most recently built, operation on weak field is fairly critical and flashovers sometimes occur. If for future building it were resolved — as is actually the case — to return to 675 V. motors connected two in series on 1 500 V., but with compensating windings, it would be well to pay some attention both to the faults of the 500 H.P. 1 500 V. nose suspended motor and to the claims of compensation.

At the very moment, therefore, when a return to 675 V. motors appears to be almost unadvoidable, there would appear to be little encouragement to adopt a voltage of 3 000, which compels us to use a type of motor which we have just condemned.

With regard to the auxiliaries on a 3 000 V. locomotive, there is no doubt that they are much more delicate than those for 1 500 V. Small 3 000 V. motors require more attention than 1 500 V. motors, and are of much more delicate construction. Though the electrical equipment itself may be similar to that on a 1 500 V. locomotive, it may be concluded that the 3 000 V. locomotive is heavier and less flexible for the same output. This last is a most important point, for the cost of the rolling stock forms some 30 to 48 % of the total outlay on an electrification project, whereas the cost of substations represents some 15 to 30 % only.

To sum up; the advantages of the 3 000 V. D.C. system are not therefore indisputable either from the economic or from the technical aspect. Some saving may be realised by its adoption, but this is small compared with the total capital outlay (some 3-7 %) and would ultimately be offset by complications in the rolling stock.

Photo Belingrodt.

Reichsbahn 1'D$_0$1' locomotive (1939), prototypes E.19 11 and 12 (Siemens). Continuous output 5 030 metric H.P. Weight 114.5 tonnes (112.19 Engl. t.). 15 000 V. Single phase.

B. 15 000 V.-16 $^2/_3$ cycles single phase system.

The single phase system at 15 000 V.-16$^2/_3$ cycles is undoubtedly the least-known traction system in France.

Apart from Mr. Auvert's experiments on the Grasse-Mouans-Sartoux section — in which single phase current was not used in the locomotive motors —, the

trials on the metre gauge railway in the South of France, or on the line from Prats de Mollo to Saint-Laurent de Cerdans, or the better-known trials conducted by the Midi in 1910, are generally disregarded or forgotten; still more so those between Lyons and Mirabel or the Haute-Vienne tramways, the latter having always been operated with this system. Similarly there are but few who have knowledge of the Ain tramway system and the Camargue railway, or of the single phase stock which runs over them. However, it is not to these last examples nor to the stock in regular services between Perpignan and Villefranche that we must turn to obtain a clear idea of modern single phase traction, since it is in this sphere of electric traction that the most substantial and recent progress has been made. To obtain an up-to-date idea of this question we must of necessity turn to the stock in other countries. For the sake of convenience, and as a logical approach, we refer not to the American systems, despite their undoubted merit, but to countries near to us such as Germany and Switzerland.

Photo C.F.F.

Lightness of fixed equipment for 15 000 V.; the approach to the main station at Zurich. Three Baeseler type double slips may be noticed.

Even an elementary knowledge of the

history of single phase traction is sufficient to confirm the pre-eminent part played by Switzerland. The Americans George WESTINGHOUSE and B.-G. LAMME may have been the first to dare to produce single phase motor vehicles, but it is to the Swiss Hans BEHN-ESCHENBURG (1864-1938) that we owe the first two single phase locomotives in the world which gave a satisfactory performance (1904-1905); the second of which was provided with series compensated motors having out-of-phase commutating fields and already embodied all the features of motors in use to-day [1]. It was Behn-Eschenburg too who, with E. HUBER STOCKAR, had the idea of selecting a special frequency which is indispensible to good commutation. In the course of the Spiez-Frutigen trials (Bernese Alps Railway) he overcame the countless difficulties in the electrification in 1910 of the first heavily trafficked railway in the world to be equipped at 15 000 V. single phase 15 — later $16^2/_3$ — cycles. During the period 1905-1910 Germany likewise took up the same problem in the electrification, among others, of the Murnau-Oberammergau, Bale-Zell to Zackingen (Wiesental and Wehratal) and Dessau-Bitterfeld lines; these were, however, very far from being attended with the same success as was achieved by the Oerlikon Company with Behn-Eschenburg in Switzerland. It is true that the Siemens Company tended towards solutions similar to those of the Oerlikon Company, but other German manufacturers (and also, at this time, the Brown-Boveri Company of Switzerland) were building extremely complicated motors which were far from offering a solution to the commutation problem. Finally, Germany had to adopt the Swiss solution, but this was only done in 1925 or thereabouts, at a time when Switzerland already possessed a

Photo Œrlikon.

Loetschberg line (Switzerland). A train of 1913. 1E1 type locomotive, series 151 to 163 (Oerlikon-S.L.M.), 2 500 metric H.P.. Two motors. Jackshaft transmission with Kando rod drive. 15 000 V. single phase.

(1) The locomotives of 1904 and 1905 are still in service in Switzerland; the one produced in 1905 in particular had undergone no changes in its electrical equipment.

considerable stock of relatively good single phase locomotives.

This preamble would appear pointless did we not feel a kind of moral obligation to re-establish a truth which German commercial propaganda, very well organised, cannot fail to obscure if we are content with a too-hasty examination. It is therefore especially of Switzerland that we shall think when we speak of single-phase traction; for while we are aware of the German achievements and admit that on the outbreak of this war they had been brought to a state of perfection more or less equal to those of the Swiss, we cannot forget that it is the small country of BEHN-ESCHENBURG which has made all the chief discoveries; the irreproachably high standard of which has been their best advertisement.

Photo Sécheron.

Modern single phase pantograph. ($1C_0C_01$ type locomotive, series 205 to 208, B.L.S. Railway, Switzerland, 1942.)

Countries which have adopted single phase current feel that its chief advantages are : simple substations, consisting mainly of a static transformer, with low maintenance costs and the use of a high voltage (15 000 volts) in the contact line; implying, other things being equal, a low current and hence a reduced weight of copper, with lower voltage drops, thus enabling the spacing of substations

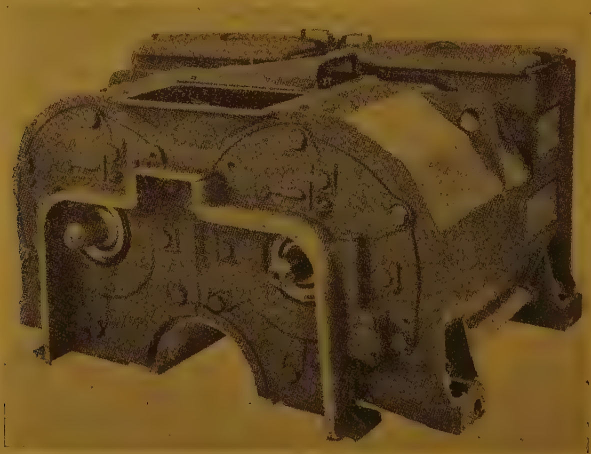


Photo Sécheron.

Twin traction motor 1 000 metric H.P. at one hour rating. Wound for 400 V. welded construction. $1C_0C_01$ type locomotive, series 205 to 208. B.L.S. Railway (Switzerland). 15 000 V. single phase.

to be increased. Briefly, in the minds of its advocates the single phase system must bring with it considerable reductions in the cost of fixed equipment.

Yet to require a series motor to operate on alternating current is rather « going against nature »; which is not slow to retaliate with bad commutation. There is thus created with the single phase motor a problem which does not exist with the D.C. motor, where the commutation, at least on full field, is always satisfactory. Manufacturers therefore concentrated either on modifying the ordinary series motor or on renouncing the series motor in order to try motors of other kinds, of which the most extensively used were the Deri repulsion and the Winter-Eichberg and Marius Latour compensated repulsion types. Since, however, motors other than series connected had shown themselves to be unsuitable for traction purposes on account of the very small range of speed variation within the allowable limits of commutation, it is the first solution — perfection of the series motor — which has been forced upon us. This was the work of Behn-Eschenburg, who discovered the reason for the bad commutation (the existence of an E.M.F., induced at very high voltage in the coil short-circuited by the brushes, under the action of a sinusoidal inducing field); analysed it and took remedial action against each difficulty in turn :

Photo C.F.F.

Swiss Federal Railways electric railcar No. Ce 2/4 701, « Jura Arrow ». Continuous output 525 metric H.P. Weight 44.4 tonnes (43.698 Engl. tons). 110 km./h. (68 m.p.h.).

1. Reduction of the frequency from 50 to 50/3 or 16.66 in order to reduce the

pulsations in the inducing field and the voltage induced in the short-circuited coil;

2. Reduction of the induced flux per pole, or, what amounts to the same thing, increase in the number of poles (for the same purpose as No. 1);

3. Addition of special poles termed « compensating poles » in series with the main poles; the ampere turns of which exactly compensate for the armature ampere turns (in order to decrease armature self-induction and neutralise its distortion);

4. Use of commutating poles which produce a field the phase angle of which is controlled by a shunting resistance.

We may set out the following conclusions as being applicable to single phase alternating current round about the year 1920 :

Advantages :

Very simple fixed equipment on railways. Substations, spaced at wide intervals, composed, in theory at least, of a static transformer. Smaller capacity required for working a given amount of traffic.

Disadvantages :

Specialised generating stations specially set aside for the production of traction current. Distribution system solely for railway purposes. Adverse effect on telephone lines.

Locomotives equipped with expensive and complicated motors with a low starting power factor.

In brief, at a time when D.C. seemed to be fully developed and only presented problems appearing easy of solution, single phase still required experiments and numerous improvements.

As we mentioned at the beginning of this article, twenty years have elapsed. During this period, have the drawbacks of single phase been offset; or, put another way, how has it been perfected ? That is, without doubt, a very wide and contentious subject, since the final outcome is that a system which presented serious difficulties has been made to function perfectly. To dwell at length on this subject would be to overstep the bounds of our article : especially since a forthcoming issue of « Chemins de Fer » describing our recent journey in Switzerland will give an exact reply to the question. Let us therefore in a few words set out the present situation as regards single phase A.C.

Firstly, as regards its effect on telephone lines. In Switzerland and Germany this has been overcome quite simply by spacing out the telephone lines, which have been removed to a distance of about one hundred metres from the catenaries. In certain cases recourse has been had to placing them in troughing. In Sweden, very nearly perfect installations have been produced (expensive to install, it is true) by the use of drainage transformers placed at suitable intervals. Be that as it may, this particular point is no longer considered to be a grave obstacle to the development of single phase lines.

As regards the single phase motor, there is a continued process of decrease in weight, improvement in commutation, increase in ventilation and increase in robustness. The motor, chief component of a locomotive, has ceased to be a delicate one. The modern single phase motor is of all-welded construction : it shows a few sparks at speeds below 7 or 8 km./h. (4 or 5 m.p.h.); under all other conditions its commutation is entirely irreproachable and altogether comparable with that of our best D.C. motors. The state of the brushes, which we have been able to check at a standstill, at starting, and at full speed, bears witness to this excellent operation. At full speed the power factor is of the order of 0.98. As a simple illustration, the motor of the 1CC1 Oerlikon type Ce 6/8 11, 14251 to 14283, which originally developed 455 H.P. at 40 km./h. (25 m.p.h.) in 1920, has been rewound accord-

ing to modern standards and now (1944) produces a continuous output of 830 H.P. at 46.5 km./h. (29.893 m.p.h.), with a saving of 47 % in the weight of copper used. The price of the modern single phase motor may be higher, but the maintenance cost is not likely to be greater than with a D.C. motor. It may be that the perusal of certain statistics (e.g. of the Reichsbahn) may lead one to believe the contrary; but it should not be forgotten that that undertaking owns a considerable number of locomotives equipped with obsolete motors (such as the E50 and E60 classes with their single 3 000 H.P. motor and very high maintenance costs) and the published figures do not give an exact indication of the true maintenance costs of modern single phase locomotives.

Finally, the most difficult problem, the frequency conversion from 50 to 16.66 cycles which makes interconnection with the national network possible, is in course of solution, not by converter groups, the efficiency of which is too low, but by a special application of mercury vapour mutators. The production of this equipment is quite within the capacity of Swiss electrical engineers.

To sum up; thanks to the ceaseless labours of its advocates, the single phase system of to-day has not only made up for lost time but on many counts is an improvement on direct current, whatever the voltage. It allows its indisputable advantages to be realised to the full : light fixed equipment, a catenary system of dimensions which depend not on electrical but on mechanical requirements and which enables aluminium-steel to be used without difficulty in place of copper. The light construction renders the general appearance of a station electrified on the 15 000 V. system much more attractive than that of our stations with 1 500 V. electrification. Surprise may also be caused by accounts we hear of voltage drops on single phase systems, yet during the whole course of our travels on locomotives in Switzerland we never had less than 14 000 V. in the line, and more often than not the voltage was 15 000 to 15 200; and this even during the ascent at 70 km./h. (43 m.p.h.) of the 27 ‰ grades on the Gothard or Loetschberg routes (it is well known that on some of our lines we have sometimes to be content with 1 100 V.).

As regards the motive power, it may be stated that, for the same output, the single phase locomotive is now lighter and much less complicated than a D.C. machine. The reduction in weight commences at the pantograph which, on Swiss locomotives of 6 000 H.P. weighs 190 kgr. (419 lbs.) (one only is sufficient to collect the whole input at 90 km./h. [56 m.p.h.]) and is continued in the underframe, the motors and the all-welded body. The equipment is entirely protected from dust in the transformer tank. Regenerative braking is possible down to a standstill. The permanent connection of the motors in parallel is particularly effective against wheel-slip. All the controller steps are running notches and the starting of freight trains by express type locomotives presents no difficulty (it is well-known that the use of express D.C. locomotives for freight working shortens the life of the resistances), a feature which largely obviates the necessity for keeping to one type for a particular duty : the Swiss habitually use their $2D_01$ locomotives on freight trains. Finally, the auxiliary motors are constructed for normal voltages (110 or 220 V.). We therefore consider that a modern single phase locomotive is scarcely more expensive than a D.C. machine; when we read in France that 80 tonne (78.7364 Engl. tons) German locomotives cost 20 % more than ours, no mention is made of the fact that for the same weight the German locomotives develop an output at least 20 % greater. There is no doubt that certain figures

published in the past on the other side of the Rhine had been purposely « inflated »; however, the fact remains that the success of single phase current, which many persist in disbelieving, is true and real. Swiss manufacturers who build both A.C. and D.C. stock have a much more extensive experience than ours, and it will be admitted that their opinion on the matter carries some weight; it does not, however, coincide with the ideas current in France. If to-day we meet in Switzerland with a 4 000 H.P. 80 tonne BB type locomotive, this is not due to larger motors but to exceptionally close design : the modern A.C. motor makes the maximum possible use of its materials; armature periphal speeds are very much higher than is customary with us (up to 64 m./s. [69.99 yd./s.] in Germany, as compared with a maximum of 45 m./s. [49.21 yd./s.] in France), and ventilation is greatly increased.

Photo Sleding.

Twin motor coach sets coupled in multiple. Netherlands Railways. 1 500 V D.C.

Whatever may be said or written, it is impossible to deny the evidence, which, when all's said and done, leads to this : a great deal more progress has been made with single phase traction in the last 20 years than with D.C. traction. Put briefly, the position is that, in A.C. traction, improvements have been made in transformers, in construction of brushgear, and in commutation, and fine control of speed has been achieved; whereas with D.C. only two really important advances have been made during the past 15 years; compensated motors and single-unit substations. To-day it may be said that there is some difficulty in deciding whether an A.C. or a D.C. locomotive is preferable : a statement which would have been utterly inconceivable 20 or 25 years ago.

It may be asked why we have just been so very much on the side of A.C., and yet why, at the beginning of this article, we said that we had very good reason to favour 1 500 V.

The reason is that, firstly, in order to be fair we had to give all the facts; we felt that we should, if only for the sake of justice, « render unto Caesar the things that are Caesar's ». This done,

we consider that France was right in adhering to the 1 500 V. system because :

(*a*) 3 000 V. presents no advantage over 1 500 V., and would prejudice the unification of our stock, and hence electrification generally.

(*b*) 15 000 V. would likewise create obstacles to unification. This reason alone is sufficient to condemn it.

(*c*) 15 000 V. is still unsuitable for suburban electrification, which forms an important part of the French proposals : the Germans themselves have electrified the Berlin suburban system at 950 V. D.C. and the Hamburg system at 1 200 V. D.C. It should be noted that in these two cases the third rail has been chosen (in the case of Hamburg the choice was not made till 1938-1939) so that eventually a 15 000 V. 16.66 cycles catenary system for the main lines might be superimposed, thus providing two distinct systems; an awkward but understandable solution of the problem.

These three points are in our view inescapable and sufficient to justify the retention of 1 500 V. D.C. They are further strengthened if we are prepared to convince ourselves that it is still susceptible of much improvement — improvement which has not taken place and which should have taken place during the last ten years or so — the broad outline of which we shall indicate towards the end of our article.

Before proceeding to a consideration of 1 500 V. D.C. let us point out that we cannot object to the fact that if we had adopted single phase it would have made us dependent on other countries in the way of licences. This would indicate both a lack of confidence on our part in our own manufacturers and also our apparent ignorance of the circumstance that a fair number of foreign patents are in use at the present time in France for our 1 500 V. system; patents of Westinghouse, General Electric, Brown-Boveri, Oerlikon, A.E.G., Secheron, Siemens, etc. origin and we cannot very well say that our locomotives are not of all-French

Photo Vilain.

2'$D_0$2' type locomotive. Series 2D2, 501 to 523 (C.E.M. Fives Lille. Brown-Boveri system). Western region, S.N.C.F. (1937). Continuous output 3 320 metric H.P. at 1 350 V. Weight 129.5 tonnes (127.455 Engl. tons).

Photo Bègue.

S.N.C.F. Western region express train, hauled by a 2'D$_0$2' type locomotive, series 501 to 523. 1 500 V. D.C. (1938).

construction, since it cannot be denied that there is a sort of international collaboration in the matter.

C. 1 500 V. D.C. system.

We feel that there would be no point in treating the subject of 1 500 V. D.C. here as we have treated 15 000 V. 16.66 cycles. The system, its fixed equipment, and its rolling stock, are well known to us. Let us, then, merely summarise its main characteristics :

Advantages :

Easy interconnection of railway distribution system with the national three-phase grid. Locomotives with motors of simple design, which are, generally speaking, fully developed and fairly cheap.

Disadvantages :

Heavy catenary system, often of necessity reinforced with feeders on account of the high currents involved. Closely spaced substations.

Complicated locomotive power circuit with uneconomical resistance starting notches.

It would seem that there is no longer much scope for progress as regards the substations themselves. However, since these must always be numerous, it is advantageous to simplify them by reducing the indoor equipment to a minimum and avoiding useless, ornate achitecture (the substations of the English « Southern Railway » are in this respect models of their kind). The use of centralised control, first applied in France on a large scale between Paris and Le Mans, is well worth extending. It is one of the factors which make for the success of the « single-unit substation » system; a system which enables both the number of converter units installed and the line

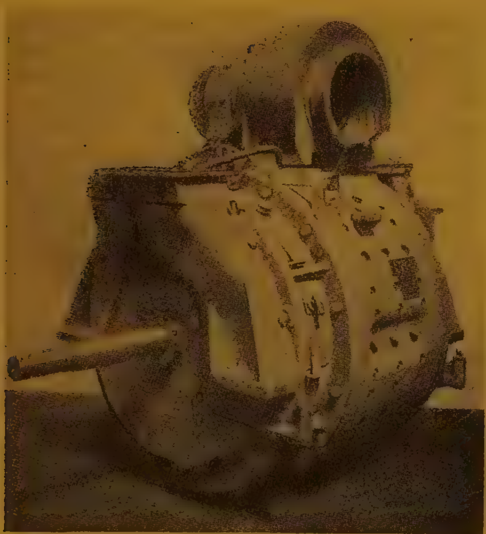


Photo C.E.M.

The most successful French D.C. motor : the HGLM 85 of the C.E.M. Continuous output 880 metric H.P. at 1 350 V. Compensating winding. This motor was advocated by Mr. Parodi for the P.O.'s E.502 locomotive.

voltage drop to be reduced to reasonable proportions. The present catenary system of the standard type designed by the P.O. in 1924 is fully developed; it would appear that little remains to be done in that direction.

Mercury arc rectifier substations are likely to be developed more and more; firstly because they lend themselves very readily to unattended automatic operation; and secondly because they have an instantaneous overload capacity which is superior to that of the rotary convertor substation. However, we should point out that the rectifier current wave contains a ripple and so causes interference both with telephone lines and with radio reception, especially if rectifiers with grid control are used (the use of resonant shunts provides a remedy for the first-mentioned). In the light of this statement it will be evident that objections raised on the same score with regard to A.C. traction lose much of their force.

It is on the motive power side that the greatest improvements still remain to be accomplished. The first to be taken in hand should be the mechanical parts; these will gain in lightness and strength by the general use of welding; they will benefit from the more frequent application of roller bearing axleboxes; and their design may well be influenced by the results of trials in Switzerland with new transmission systems which are likely to lead to the elimination of nose-suspended motors. On the electrical side, it is important that all future 1 500 volt stock should be equipped with motors which commutate satisfactorily under all operating conditions and with equipment providing a degree of flexibility equal to that of single phase stock. This result has for many years been within our power to accomplish without complication as regards present day locomotives. The solution consists simply in adopting the series-compensated motor used on single phase locomotives and supplying it with direct current. Other things being equal, this will result in a shaft output 10 % higher than with single phase current, with better commutation and less wear. Moreover, the complete compensation provided on these motors would enable them to be worked with a very high value of field shunt (80 %); at the same time the problem of flexibility would be resolved and the necessity for depending on resistances would be largely eliminated. There is no doubt that this is the direction in which the most interesting progress has still to be made with the 1 500 V. system [1]. Evidently the adoption of this method also demolishes one

[1] We do not, of course, forget the interest attaching to the production of improved metadyne equipments.

argument against A.C., since the cost of motor construction is the same in both cases. We feel however that the cost will be more usefully expended in this way than in changing the methods of streamlining every ten locomotives or so.

We are absolutely convinced that, if these few principles are observed, our 1 500 V. system is able to meet our present and future needs with as much success as any system imaginable. It will always be superior to single phase as regards better suburban working, and to 3 000 V. as regards simplicity of stock. Finally, unification will enable us to establish very large groups of stock, which will lead to a better use factor and a reduction in total numbers required. We feel that there is no point in our dwelling on the difficulties which would have been occasioned by the points of junction of lines electrified on different systems; mere consideration of such junctions as those between single and three phase, or single phase and 3 000 V. D.C. (Switzerland - Italy); 1 500 V. D.C. — three phase (France - Italy), etc.; should lead us to reject any system other than 1 500 V. in France. We have to convince ourselves that, far from being a makeshift, 1 500 V. D.C. is destined to become more and more worthwile; especially if, for the maximum benefit to the users, the amortisation of the fixed equipment is rendered easier by the provision of a frequent and rapid train service. It is fortunately the case that this question of service frequency, which is in our opinion one of the safest defences against road competition, is also that to which electrification best lends itself; a frequent train service is only economically obtainable with electric traction, and electric traction becomes progressively more economical as the service frequency is increased. Why then not make full use of this coincidence ?

Photo C.M.P.

Sceaux line, near Bagneux-Pont Royal (Paris Metropolitan Railway). 1 500 V. D.C.

Conclusion.

It is, then, time to say that 1 500 V. should in future be considered as the sole system of electrification to be adopted in France ? Let us say at once that the only system worthy of consideration in its stead would be the single phase 50 cycles (industrial frequency). Thanks to the experiments of Kandô in Hungary, and the Reichsbahn on the Höllenthal line (Fribourg-Titisee [Neustadt]), it seems that the dream which the engineers of 1904 could not realise may be on the point of becoming reality. If in the future we have to electrify lines of low traffic density but with steep grades and with but few points of contact with the general 1 500 V. system, 50 cycle single phase should hold our complete attention.

We may perhaps be reproached for not having set up in this article numerous and highly imposing columns of figures. If such had been our wish, we could easily have reproduced, for example, those emanating from the conference held at the Institute of Industrial Engineers, Barcelona on the 12th June 1945 under the auspices of the French Institute at Barcelona. Yet what use would they have been; for :

« These figures appear to lead inevitably to the conclusion that, so far as large scale electrification is concerned, single phase and direct current both commit the Railway System to capital outlays which are almost, if not entirely equal... The cost is the same whichever kind of current is chosen. »

We have not sought here to prove otherwise. We have merely wished to indicate certain factors regarding electrification systems which appear to us to have been omitted in other articles on the subject, and to lay stress upon these two facts : firstly, that 15 000 V. single phase 16.66 cycles would in all circumstances be preferable to 3 000 V. D.C. (in the ultimate issue this latter has the sole advantage of causing less upset to practices originating with 1 500 V.) and secondly, that if we reject 15 000 V. 16.66 cycles it cannot be for the reasons commonly put forward.

Utilization of new types of material in buildings.(*)

(From the Railway Engineering and Maintenance, November, 1947.)

Generally speaking, there are relatively few building material items presently available for extensive and economic use which can properly be classed as distincly new types of materials. Insofar as railroad building construction and maintenance is concerned, however, there are a number of materials which can be called new types because they have not heretofore been generally used.

The subject matter of this report should not be considered as completely covering the entire field of recent developments, effort having been made to limit the scope of the report to only such items as appear to possess qualities making them desirable for use by the railroads. Ten general classifications of new types of materials have been considered.

Glass products.

Problems of providing maximum light transmission and over-all pleasing appearance are being solved successfully and economically by the use of corrugated glass sheets, either of wire glass or plain glass, and by the use of glass blocks.

Corrugated wire glass is applicable for use as siding, side panels of saw-tooth type roof construction, enclosure material for elevator shafts and the like, and roofing in skylight, marquise and canopy construction. Plain corrugated glass may be used for interior partitions and decorative glass panels and can be either smooth finished or sandblasted on one or both sides.

Sheets of corrugated glass can be applied directly to structural framework, can be made water-tight, and can be used readily in conjunction with corrugated metal, corrugated asbestos or other types of siding and roofing.

Where the simultaneous provision of light transmission and ventilation is not mandatory, structural glass blocks are particularly valuable for use in roundhouses, machine shops, etc. These blocks readily take the place of conventional windows and, when used in connection with construction of exterior walls, have the added advantage of substantially increasing light transmission without materially disturbing the heating factors of the building as a whole.

Glass blocks are available in several sizes and designs, the several designs being for the purpose of meeting various problems and requirements of light transmission. From an architectural standpoint, glass blocks can be worked into attractive panel arrangements. Panels can include clear glass window sash when ventilation is required or small areas of visibility are desirable. Glass blocks are quite strong, but not load bearing, and are not subject to ordinary breakage. If broken, they are easily replaceable. Panels are readily cleansed by wetting, brushing and washing down with a hose.

Structural glass.

Structural glass exterior and interior finish is rapidly gaining in favor among

(*) Report of Committee at the Chicago meeting on September 16-18, 1947, of the American Railway Bridge and Building Association.

designers. There are two types of this kind of glass; sheets of polished homogeneous material, and precast, load-bearing concrete blocks faced under pressure with polished structural glass. These products provide the designer with a means of achieving extremely attractive decorative effects, both exterior and interior, without sacrifice of economy. They are durable, not easily damaged, and are readily maintained and cleansed. The homogeneous sheets are quite suitable for dado and wainscot purposes and prefabricated sheets can be used for partitions and inside wall finish.

For construction requiring special treatment because of heating or air-conditioning, insulating glass units can be used satisfactorily. These units are formed by sealing dehydrated air between two or more panes of window or plate glass. The framework for these units must be of rigid design to prevent damage to them. Resistance of the units to the transmission of heat is high.

Heat-absorbing glass.

Heat-absorbing glass is rapidly coming into general use in various types of shop buildings, storehouses, and other buildings. This is a blue-green colored glass, that absorbs most of the sun's heat rays, admits an adequate amount of light, and substantially reduces glare and eyestrain. It is available in thicknesses of $^1/_8$ in. and $^1/_4$ in. Ordinary glass $^1/_8$ in. thick transmits about 87 per cent heat, while the same thickness of heat-absorbing glass transmits only 34 per cent; similarly, $^1/_4$ in. thick ordinary glass transmits 83 per cent heat, while the same thickness of heat-absorbing glass transmits 21 per cent. Heat from the sun is absorbed by this type of glass, thus raising the temperature of the glass until the point is reached where re-radiation equals the heat supplied by the sun. The absorbed heat is re-radiated from both surfaces of the glass, and it is assumed that one half goes to the outside and the other half to the inside.

Sun screens.

Although not a glass product, mention should be made here, apropos light and ventilation, of a relatively new window screening. This material was developed to provide a shading device, permitting passage of light and ventilation and at the same time protect against the direct radiant heat from the sun.

This sun screen, built on the principle of the venetian blind, makes it possible to achieve 80 to 90 per cent effective radiant-heat window insulation with practically no interference to ventilation, light or view. Louvres, of minute width bronze, 17 to the inch, are set at a fixed angle determined to give maximum efficiency with minimum interference to view and light. The sun screen is installed in same manner as conventional window screening. Its use is particularly indicated in conjunction with air-conditioning installations.

Paint products.

Silicone resin paints have entered the field of heat-resisting and moisture-resisting materials. Some blends of the material will withstand continuous exposure to temperatures as high as 500 deg. F. and are ideal for such applications as ovens, smokestacks, exhaust manifolds, radiators and the like. Other blends of silicone resin paints are highly satisfactory for heat and moisture-resistant insulations for electric motors, high and low-temperature greases, heat-stable fluids and anti-foam compounds.

Water-base cement-binder paints have come to the forefront in the painting of masonry where conditions are such that oil-base paints cannot be successfully applied and where it is considered desirable to re-paint every three or four years. These paints are normally furnished in

powder form, to which water is added immediately prior to use.

As a rule, the ingredients of water-base cement-binder paints are more or less the same, and all of the many brands available will produce about the same results. An attractive decorative coating with three or four year's life can be secured on brick, stucco, concrete or tile walls by use of these paints. Many claims have been made with respect to the water-proofing qualities of these materials, but in most instances such claims are somewhat exaggerated and the materials will not stand up very well in most areas when used under abnormal conditions.

Oil-base cement-binder paints are actually normal oil paints with portland cement added. Such paints have been on the market for a number of years, but only recently have been used extensively. In normally dry climates of low humidity, oil-base cement-binder paints can be used successfully for the painting of masonry, provided the masonry has been properly sized with an alkali-proof sealer before painting.

Painting masonry walls.

Masonry walls painted with oil-base cement-binder paints will not require repainting as often as when water-base type paints are used. These products have about the same water-proofing qualities as water-base products. One prominent characteristic of the oil-base product is that it can be applied with lasting and satisfactory results directly to new galvanized metal without the necessity of allowing the metal to weather or of applying a special under-coater.

There have been some recent developments in fire-resistive coatings, although insufficient test data is presently at hand upon which to base recommendations for their use. Oil-base cement-binder paints have some value as fire-resistive coatings, particularly when the final coat is sanded. Several other special products recently placed on the market appear to have characteristics which tend to resist fire exposure.

Considerable progress has been made in the development of rust-inhibitors for use on metal structures. While most of these products were available before the war, there has been a decided improvement in their characteristics. These materials, of which there are a number equal in quality, can very often be used to good advantage.

Plastic coating.

While not exactly a paint, a war-born plastic paint-like product is now ready for general distribution. This product is a brushless plastic finish for applying to furniture, walls, floors, cabinets, wood trim, linoleum, etc. The material is wiped on with a cloth, touch dries in ten minutes and dries hard in six hours. No conclusive test data is yet available as to its lasting qualities or economy.

Closely associated with the subject of paint and painting is the recent development of a special pre-treatment for galvanized steel sheets. This pre-treating process forms a phosphate film on zinc surfaces, making it possible to apply paint to zinc-coated materials successfully without having to make allowance for a suitable weathering period.

Pre-cast concrete products.

Concrete blocks are being used extensively as substitutes for brick and hollow tile and in lieu of poured-concrete construction. Their properties permit savings in steel and formwork and afford economies in heating. The blocks have a high sound insulation value and are easy of installation and later removal, if desired.

There are two types of blocks; those similar in design to hollow tile, requir-

ing mortar joints, and those with interlocking edges. The interlocking type is laid up without mortar joints, rigidity of the wall being achieved by filling the interstices of the blocks with concrete at regularly spaced intervals.

Leakage through walls, due to porosity of the blocks, generally requires the application of some type of waterproofing agents to the finished exterior wall surfaces.

Pre-cast concrete roof slabs and beams of various types and designs have been on the market for a number of years but have only recently come into general usage. For permanent and fire-safe construction, use of these items may well be indicated, particularly for large projects where sizes can be made more or less standard.

Wash fountains and shower stalls of pre-cast concrete are in wide use in industrial wash and shower-room areas where a great many people have need for washing facilities at about the same time. These items are economical, durable, easy to keep clean and presentable, and are not difficult of installation.

The waiting room of the Erie's new passenger station at Akron, Ohio, is an example of the use of the latest type of building material in railway structures.

Many large industries have successfully located central pre-casting plants for small concrete buildings. Forms are used over and over and pouring of the concrete is carried on under rigid control, thereby insuring soundness of construction. When small buildings, such as watchmen's huts, telephone booths,

etc., are needed, they can be promptly shipped to the location required and erected quickly without the services of skilled labor, a feature that is becoming increasingly important.

Cement-asbestos products.

Corrugated sheets of cement-asbestos are suitable replacements for corrugated metal and can be satisfactorily used for most types of railroad building construction requiring corrugated metal. Some of their advantages are high resistance to weather and temperature variations, ease and speed of construction, and non-combustibility. Flat sheets of cement-asbestos have substantially the same characteristics as corrugated sheets, although they have much less strength.

Rigid asbestos shingles for building work are coming into wide use, both for roofing and for siding, there being a great variety of such shingles available. Use of shingles is indicated for remodeling the exteriors of frame buildings and for the replacement of wood shingles on roofs. Very attractive results can be obtained by the proper and considered application of shingles of this type.

Cement-asbestos lumber.

Recent developments in the use of cement-asbestos lumber seem to indicate solution of the problem of providing a movable and salvable partition wall, and at the same time of achieving an appearance of stability and permanence. Partition units are formed by the use of standard studding to which the cement-asbestos lumber is attached by special fastenings. The studding in the wall units is not attached to the floor or ceiling, special floor and ceiling channels being used for the purpose of maintaining the units in place.

In addition to being almost 100 per cent salvable, the wall units have the advantages of providing an easily maintained surface, good resistance against the passage of sound, resistance against fire exposure, and a surface free of projections. Their use is indicated where space requirements in offices and similar places are subject to relatively rapid changes.

Impregnated cement-asbestos sheets are indicated for use where especially high-resistance to moisture, chemical fumes and gases is desired. Both surfaces of the sheets are treated with a bituminous compound, which acts as a seal against attack by destructive agents.

Structural aluminium products.

Increased use of structural aluminium products is the result of efforts by designers to reduce weight of construction without impairment of strength, to achieve improvement in resistance to corrosion, and to present a pleasing architectural appearance. The aluminium used in the manufacture of structural shapes, etc., is in the form of alloys of various characteristics. In selecting aluminium products for use, therefore, consideration must be given to the purpose for which the aluminium is to be used.

Alloys are produced in many forms by drawing, extruding, rolling and forging. Structural shapes, such as angles, beams, tees and channels, and plates of relatively large size, are available. Also available are special architectural shapes, such as handrails, pilasters, mouldings, cornices and fascia, copings, thresholds, etc.

For roofing and siding.

Aluminium sheet is also manufactured for use as roofing and siding. Roofing is made in the interlocking-shingle type, flat type with pre-formed joints, or corrugated-sheet type. Siding is available in flat or corrugated sheet, drop siding, or clapboard type. Some of the corru-

gated aluminium roofing and siding, however, is of too light a gage to be recommended for industrial use. In addition, aluminium is not recommended for use around engine terminals due to the attack made upon it by locomotive gases.

After continued exposure, the bright natural finish of aluminium becomes somewhat dull. To overcome this dulling, a special finishing process has been developed, which augments the natural film of aluminium oxide, thereby protecting the metal against weathering, etc. In addition, the finish permits the adding of color if desired.

Floor finishes.

Flexible floor coverings in general use at present include cork tile and sheet, asphalt tile, rubber tile, and linoleum, all of which have several disadvantages along with their several advantages. A new type of flexible floor covering made from both plasticized and unplasticized vinyl resin is becoming available in the form of floor tile or rolled floor sheet. Vinyl-resin floor coverings have all the advantages of other types of flexible coverings, with the added advantages of high strength, durability and resistance to attacks by greases, acids, alkalis, etc. The coverings can be furnished in a wide range of colors and their maintenance is especially easy due to their smooth finish and resistance to greases and other such agents.

In the maintenance of concrete floors and, in some instances, wood floors, use of one of several asphaltic-mastic type of floor re-surfacers may be indicated. These products, sold under various trade-names, are more or less similar in characteristics and performance. They are useful in cases where concrete floors have become badly worn, pitted or broken and patch repairs are considered advisable instead of overall floor replacement. The material sets up to a fairly hard finish, is not difficult to place, and will stand a great deal of hard usage.

Admixtures.

Several types of concrete admixtures have been developed in recent years for use in new concrete floor construction. These admixtures are for the purpose of securing durable, impervious and non-dusting surfaces for concrete floors subject to heavy traffic. Good resistance to attacks by destructive agents is also secured by use of certain types of admixtures.

A new type of steel floor plate appears to have excellent possibilities for floor work. These plates are 12 in. square, flanged down on all four sides, and have many teeth-like anchors stamped from the wearing surface to provide bond to the concrete. Use of these plates is particularly desirable where floors are subjected to heavy and sustained travel.

Interior wall finishes.

Enameled steel tile is available in baked enamel and porcelain enamel finish and a variety of colors. The baked enamel finish can be marred by contact with sharp objects. The porcelain enamel finish will resist harsh treatment and its surface will not craze or crack. The tile is particularly useful for corridors, washrooms and toilets, dining rooms, and similar areas.

Cloth-back wood veneer in 40 odd types is being successfully used for decorative purposes where an appearance of luxury and distinctive wall treatment is desired. The product is genuine paper-thin wood veneer glued under heat and pressure to cotton sheeting with water-resistant adhesive. The sheets can be hung to provide almost any design pattern desired.

Cloth-backed glass is very similar to cloth-backed wood veneer and can be used in much the same manner as the

wood veneer. Thin sheets of glass are bonded to fabric backing and are then cut into small squares or rectangles. The material is available in 25 or more colors and 4 types — opaques, flat mirror, rolled pattern mirror, and metallic. Corners with a minimum radius of 5 in. may be turned without difficulty. The product is resistant to every surface attack to which sheet glass is resistant.

Sheet plastic on plywood provides a finishing material readily adaptable for many interior uses. The product is formed by fusing liquid plastic to plywood by heat treatment. The surface of the material is hard, smooth and polished and is easily maintained. The material should not be used in locations where it will be subjected to abnormal abuse and attacks by vandals.

Glazed-surface wood veneer sheets are substantially the same as sheet plastic on plywood and, therefore are suitable for the same purposes. The glazed surface sheets are not so durable as the plastic sheets, but they can be used to good advantage in numerous locations where they are not subject to abuse other than normal cleansing operations.

Although fibre or composition ceiling blocks and wallboard panels and planks have been available for some years, use of these materials for interior re-decoration and modernization has not been general until recently. These products lend themselves readily to interior refinishing work at relatively low costs. Ceiling blocks, both acoustic and non-acoustic, present excellent means for refinishing ceilings. Attractive wall treatment can be achieved by use of wallboard panels or planks.

Metal buildings.

War demands developed considerable usage of pre-fabricated metal buildings. Buildings of many sizes and several types of pre-fabrication are available and have proved to be relatively economical and entirely satisfactory. Among their chief advantages are rapidity and ease of construction and adaptability to ready removal from place to place without loss of any part except foundation.

These buildings are generally either of the frameless type, in which the flanged panels are the load-carrying members, or the light steel framing type, in which the framework is covered with light-weight metal sheets. All parts of the buildings are metal, although wood sash and doors can be used if desired. Various combinations of floor plan can be achieved without difficulty and the over-all appearance is not unpleasing.

Metal roofing.

Pre-fabricated steel roof decks of a number of types are now being used to good advantage under normal conditions. These decks can be substituted readily for pre-cast concrete and gypsum and usual wood decks and have the advantage of holding the roof framing costs to the minimum. One relatively new type of metal deck consists essentially of flanged interlocking steel panels with the flanges turned down. Insulating board and built-up roofing can easily be applied over metal decks.

Around terminals and other locations where corrosion and abrasion must be considered, asbestos-bonded steel sheets have particular usefulness. These sheets are made of steel or iron, coated with zinc in which asbestos fibres are imbedded while the zinc is molten. Final treatment consists of an application of a bituminous saturant under pressure.

Pre-cast gypsum products.

One pre-cast gypsum product — sheet-rock — has long been in use for interior partition work. Out of sheet-rock has been developed solid partition units and gypsum roof plank, which are merely

sheets of sheet-rock glued together under heat and pressure.

Partition units are available in tongue-and-groove type or shiplap type, in thicknesses of 1 in. and 1 1/2 in. These units are particularly useful for non-load-bearing partitions and where height and length of partitions are not excessive. Wood bucks and top and bottom plates laid flat against the panels add considerable strength and result in a pleasing appearance.

Roof units.

Gypsum roof units are two-ply, three-ply, or four-ply, with the sheets in the plies being placed off-center to form ship-lap edges. Thicknesses are 1 in., 1 1/2 in. and 2 in. and the units are treated with a water-resistant coating after the sheets have been glued together. The units will carry almost the same superimposed roof load as wood sheathing of comparable thickness, and are adequate replacements for wood as the decking for built-up roofs. They afford the added advantages of speed and economy in placing, due to their large size, and the further advantage of being rot-proof and vermin-proof.

Conclusion.

The necessity for brevity in this report has made it impossible to do more than touch briefly on the many new building material items which can well be used in various types of railroad work. Conspicuous by their absence are remarks apropos to the many new developments in the heating, ventilating and lighting fields. There have been numerous new items and advances in these fields in the last few years, most of which are worthy of consideration for use in railway freight and passenger stations, shops, enginehouses, office buildings, service buildings and all types of roadway structures.

As partial basis for the substance of this report, reference was made to the report of the Building Committee of the American Railway Engineering Association, published by the association in Bulletin 463 of December 1946. Readers hereof desiring more detailed information about many of the products mentioned are referred to this A.R.E.A. committee report.

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OFFICIAL INFORMATION

ISSUED BY THE

PERMANENT COMMISSION

OF THE

International Railway Congress Association.

Meeting of the 14th February 1948, of the Permanent Commission.

The Permanent Commission of the International Railway Congress Association met on the 14th February 1948, in the Belgian National Railways' Head Offices at Brussels.

* * *

Mr. Delory, *President*, opening the meeting welcomed the many personalities present.

Then he asked the meeting to approve the Minutes of the last meeting held at Lucerne on the 28th June 1947.

Mr. Dechitch *(Jugoslavia)*, speaking in the name of his country, Poland, Czechoslovakia and Bulgaria recalled the request made at Lucerne to put the Association in a position to make possible the linking with U.N.O. and asked that this question, which involves the examination of the situation of Spain within the Association, should be added on the agenda of the meeting.

The President confirmed the reply made at Lucerne, which mentioned that the examination of the situation of Spain depended on the result of the written ballot prescribed by art. 4 of the Rules and Regulations of the Association and that the application of art. 4 had to be applied for by a certain number of members of the Permanent Commission.

As this last condition had not been fulfilled, the consultation prescribed by art. 4 could not have been done and therefore it had not seemed advisable to call a meeting of the Special Commission, which had to state the position of the Association with regard to the question of linking with U.N.O., as mentioned in the reply of the Permanent Commission to the Jugoslav, Polish, Czech and Bulgarian members.

The President confirmed that the written ballot prescribed by art. 4 will only take place when the Secretariat of the Permanent Commission, will be in possession of positive requests from a certain number of members of the Association.

Following this statement, the Jugoslav, Polish and Czech members stated that they will only attend the present meeting as observers.

The President dealt next with the changes in the Permanent Commission since its last meeting and gave particulars of the steps taken to select the personalities for the mandates available.

The Meeting then elected the following as members of the Permanent Commission :

Mr. Rafaël Luna, Engineer, General Director of the Argentine State Railways;

Mr. Pedro Pablo Martin, Engineer, General Inspector of the Commercial Operation, Under-Director of the Argentine State Railways;

Mr. M. Devos, General Manager, Belgian National Light Railways, replacing Mr. Jacobs, who has resigned;

H.E. El Sayed Gawdat Bey, Under-Secretary of State, Egyptian Ministry of Communications, to replace H.E. Mohamed Abdel Khalek Saber Bey, former Under-Secretary of State at the same Ministry;

Sir Gilmour Jenkins, Permanent Secretary, British Ministry of Transport, will take the place of Sir William V. Wood who has resigned;

Mr. V. M. Barrington-Ward, C.B.E., D.S.O., Member of the British Railway Executive, will replace Sir James Milne, who has retired.

The Meeting agreed with the President's proposal to elect Sir Gilmour Jenkins as member of the Executive Committee to take the place of Sir William V. Wood (*).

* * *

Mr. Ghilain, *General Secretary*, then gave the results of the written ballot concerning the Art. 17 of the Rules and Regulations. The results were published in the *Bulletin of the Association* for January 1948.

The General Secretary acquainted also the meeting with a communication relating to the organisation at Lisbon, in 1949, of a meeting of the enlarged Permanent Commission.

It was agreed to held this meeting during the Spring of 1949, in accordance with the wish expressed by the Minister of Communications of Portugal.

The Meeting then selected the 3 following questions for discussion at the Lisbon meeting and choice was made of the countries to be requested to appoint the reporters.

Section I.
WAY AND WORKS.

Question :

a) Mechanization of the maintenance and renewal of the Permanent way.

b) Recent improvements relating to reinforced concrete and prestressed concrete sleepers.

Results obtained.

c) Recovery and strengthening of metal bridges which have reached the theoretical limit of safety.

Reporters from :

France, for the French report;

Great Britain, for the English report, and

Portugal, for the special report.

Section II.
LOCOMOTIVES AND ROLLING STOCK.

Question :

Electric locomotives for fast trains (75 m.p.h. and over). Discussion of adopted and projected types.

1) Arrangement of the axles;

2) Type of axle drive :

a) motor suspended from the nose;

b) flexible transmission;

3) Electric motor characteristics;

4) Braking.

(*) From information received after the meeting of the Permanent Commission, it seems that this nomination might be subject to modification.

Reporters from :

Switzerland, for the French report,

South Africa, for the English report, and

Italy, for the special report.

SECTIOH III.

WORKING.

QUESTION :

Transport of miscellaneous goods.

Concentration in a certain number of selected centres (stations) of miscellaneous traffic, transport by rail between centre-stations, by road or rail between the originating points and the nearest centre-station, and also to the last centre-station near the destination.

Interest of the scheme for the conveyance of goods traffic. Organisation of the station-centres and of the collection and delivery services.

Financial results of the scheme.

Reporters from :

Belgium, for the French report;

India, for the English report, and

Portugal, for the special report.

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The Meeting adopted a suggestion of the GENERAL SECRETARY regarding the composition of the delegations. After the reading of a letter from Mr. DA COSTA COUVREUR, Portuguese member of the Permanent Commission, giving a proposed programme of the Lisbon Session, it was decided to limit to 4 days the duration of the works.

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A proposal of alteration of the Rules and Regulations made by the « Compagnie du Chemin de fer Métropolitain de Paris » in order to allow for such railways the determination of the number of delegates to the Congress according to the importance of their passenger traffic and not as at present following their mileage which is usually small, was brought to the notice of the meeting.

It was decided to study this question at a next meeting, as the Statutes of the « Chemin de fer Métropolitain de Paris » are on the eve of being altered. Meanwhile the examination of the possible effects of this proposal will be done.

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The statements of receipts and expenditure for the year 1947 were approved by the Meeting as well as the provisional budget for 1948. To meet the financial needs, the Meeting decided that the rate of the variable subscription will remain in 1948 the same as was fixed for 1947, i.e. 0.15 gold-franc per kilometre. It is recalled that the maximum of the variable subscription may not exceed the third of a gold-franc per kilometre according to the Rules and Regulations.

The report drawn up by the auditors on the verification of the accounts of the 14th. Session, i.e. for the years 1938 to 1947, was submitted to the Meeting and approved.

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Information was also given about the changes which occurred in the membership since the previous meeting of the 28th June 1947.

The International Railway Congress Association at present includes 35 Governments, 9 Organisations and 138 Administrations with a total mileage of 450 000 km. (280 000 miles).

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The meeting ended after the examination of various points regarding the activities of the Association since the last meeting of the Permanent Commission.

The General Secretary, P. GHILAIN.

The President, DELORY.